Plan for today

- Simplex Method: Algebraic Form
  - Example
- Simplex Method: Tabular Form
  - Example 1
  - Example 2
- Shadow Prices

SIMPLEX METHOD

Simplex Method: the complete algorithm

- **Initialization**
  - transform the original LP into the augmented LP, determine basic and non-basic variables
  - Rewrite constraints in proper format:
    - one basic variable on the LHS with coefficient 1,
    - constants and non-basic variables on the RHS and
    - RHS constant should be non-negative
  - Rewrite objective function in proper format: contains only non-basic variables

- **Iteration**
  1. select the entering variable: the variable with the largest positive coefficient in the obj. function
  2. select the leaving variable **(min-ratio test)**: the first basic variable that reaches 0 as the value of the entering variable increases
  3. express the entering variable as a function of non-basic variables using the constraint that identified the leaving variable
  4. use the expression obtained at step 3 to replace the entering basic variable in the objective function and in all constraints

- **Termination: Optimality Test**
  - if every variable in the objective function is with a negative coefficient, then no entering variable can be found ⇒ **the optimal solution has been found**
  - if the entering variable can be increased to infinity without driving any other basic variable to below zero, then no leaving variable can be found ⇒ **the problem is unbounded**

SIMPLEX METHOD: ALGEBRAIC FORM is what we have seen so far

… is the form that uses symbols and rewrites equations

SIMPLEX METHOD

Yet another example

$$\max Z = 6x_1 + 5x_2$$
$$2x_1 + x_2 \leq 4$$
$$3x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

Initialization

Original LP

$$\max Z = 6x_1 + 5x_2$$
$$2x_1 + x_2 \leq 4$$
$$3x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

Augmented LP

$$\max Z = 6x_1 + 5x_2$$
$$2x_1 + x_2 + x_3 = 4$$
$$3x_1 + x_2 + x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Initialization

$$Z = 6x_1 + 5x_2$$
$$x_3 = 4 - 2x_1 - x_2$$
$$x_4 = 5 - 3x_1 - x_2$$

Basic variables: $x_3 = 4, x_4 = 5$

Non-basic variables: $x_1 = x_2 = 0$

$Z = 0$

Iteration

Iteration 1:

$$Z = 6x_1 + 5x_2$$
$$x_3 = 4 - 2x_1 - x_2$$
$$x_4 = 5 - 3x_1 - x_2$$

Basic variables: $x_3 = 4, x_4 = 5$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Entering variable: x_1
Leaving variable: x_4

$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$
$$x_3 = 4 - 2\left(\frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4\right) - x_2$$
$$= \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

$$Z = 6\left(\frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4\right) + 5x_2$$
$$= 10 + 3x_2 - 2x_4$$

$$Z = 10 + 3x_2 - 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$
$$x_3 = \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

Basic variables: $x_1 = \frac{5}{3}, x_3 = \frac{2}{3}$
Non-basic variables: $x_2 = x_4 = 0$
$Z = 10$

Iteration

Iteration 2:

$$Z = 10 + 3x_2 - 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$
$$x_3 = \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

Basic variables: $x_1 = \frac{5}{3}, x_3 = \frac{2}{3}$

Non-basic variables: $x_2 = x_4 = 0$

$Z = 10$

Entering variable: x_2

Leaving variable: x_3

$$x_2 = 2 - 3x_3 + 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}(2 - 3x_3 + 2x_4) - \frac{1}{3}x_4$$
$$= 1 + x_3 - x_4$$

$$Z = 10 + 3(2 - 3x_3 + 2x_4) - 2x_4$$
$$= 16 - 9x_3 + 4x_4$$

$$Z = 16 - 9x_3 + 4x_4$$
$$x_2 = 2 - 3x_3 + 2x_4$$
$$x_1 = 1 + x_3 - x_4$$

Basic variables: $x_1 = 1, x_2 = 2$

Non-basic variables: $x_3 = x_4 = 0$

$Z = 16$

Iteration

Iteration 3:

$$\boxed{\begin{aligned} Z &= 16 - 9x_3 + 4x_4 \\ x_2 &= 2 - 3x_3 + 2x_4 \\ x_1 &= 1 + x_3 - x_4 \end{aligned}}$$

Basic variables: $x_1 = 1, x_2 = 2$
Non-basic variables: $x_3 = x_4 = 0$
$Z = 16$

Entering variable: x_4

Leaving variable: x_1

$$\begin{aligned} x_4 &= 1 + x_3 - x_1 \\ x_2 &= 2 - 3x_3 + 2(1 + x_3 - x_1) \\ &= 4 - x_3 - 2x_1 \end{aligned}$$

$$\begin{aligned} Z &= 16 - 9x_3 + 4(1 + x_3 - x_1) \\ &= 20 - 5x_3 - 4x_1 \end{aligned}$$

$$\boxed{\begin{aligned} Z &= 20 - 5x_3 - 4x_1 \\ x_2 &= 4 - x_3 - 2x_1 \\ x_4 &= 1 + x_3 - x_1 \end{aligned}}$$

Basic variables: $x_2 = 4, x_4 = 1$
Non-basic variables: $x_1 = x_3 = 0$
$Z = 20$

Termination

$$Z = 20 - 5x_3 - 4x_1$$
$$x_2 = 4 - x_3 - 2x_1$$
$$x_4 = 1 + x_3 - x_1$$

Basic variables: $x_2 = 4, x_4 = 1$
Non-basic variables: $x_1 = x_3 = 0$
$Z = 20$

No entering variable, so terminate
Optimal solution is $x_1^* = 0, x_2^* = 4$

Initialization

Original LP

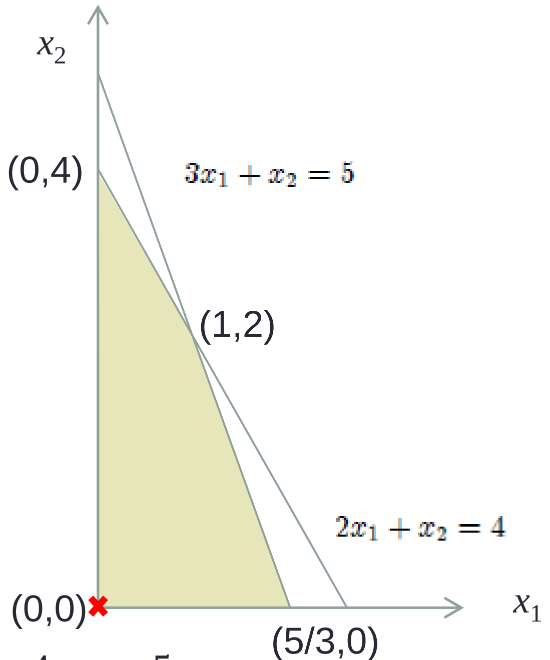
$$\begin{aligned}&\max Z = 6x_1 + 5x_2\\&2x_1 + x_2 \le 4\\&3x_1 + x_2 \le 5\\&x_1, x_2 \ge 0\end{aligned}$$

Augmented LP

$$\begin{aligned}&\max Z = 6x_1 + 5x_2\\&2x_1 + x_2 + x_3 = 4\\&3x_1 + x_2 + x_4 = 5\\&x_1, x_2, x_3, x_4 \ge 0\end{aligned}$$

Initialization

$$\begin{aligned}&Z = 6x_1 + 5x_2\\&x_3 = 4 - 2x_1 - x_2\\&x_4 = 5 - 3x_1 - x_2\end{aligned}$$



Basic variables: $x_3 = 4, x_4 = 5$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Iteration

Iteration 1:

$$Z = 6x_1 + 5x_2$$
$$x_3 = 4 - 2x_1 - x_2$$
$$x_4 = 5 - 3x_1 - x_2$$

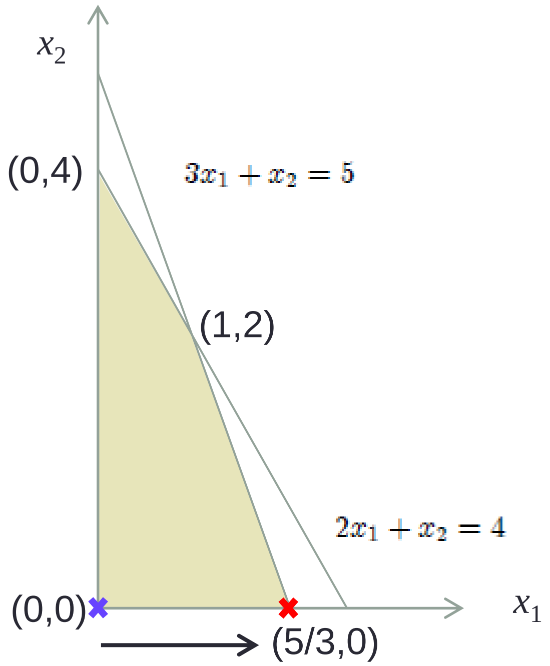
Basic variables: $x_3 = 4, x_4 = 5$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Entering variable: x_1
Leaving variable: x_4

$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$

$$x_3 = 4 - 2\left(\frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4\right) - x_2$$
$$= \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

$$Z = 6\left(\frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4\right) + 5x_2$$
$$= 10 + 3x_2 - 2x_4$$



$$Z = 10 + 3x_2 - 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$
$$x_3 = \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

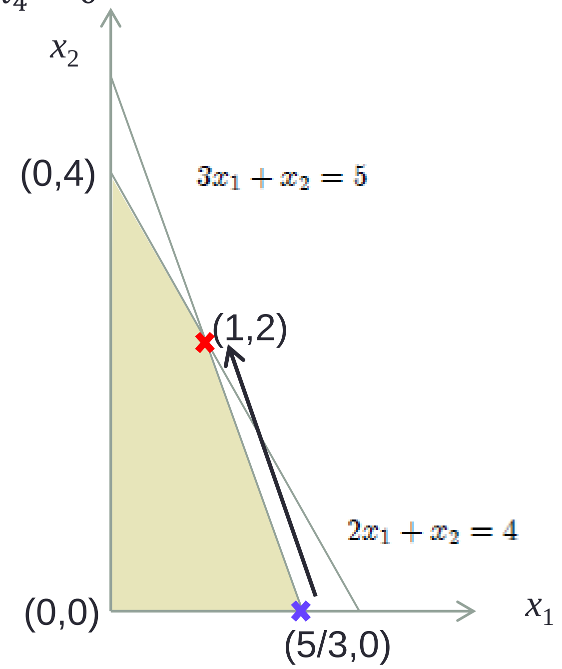
Basic variables: $x_1 = \frac{5}{3}, x_3 = \frac{2}{3}$
Non-basic variables: $x_2 = x_4 = 0$
$Z = 10$

Iteration

Iteration 2:

$$Z = 10 + 3x_2 - 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_4$$
$$x_3 = \frac{2}{3} - \frac{1}{3}x_2 + \frac{2}{3}x_4$$

Basic variables: $x_1 = \frac{5}{3}, x_3 = \frac{2}{3}$
Non-basic variables: $x_2 = x_4 = 0$
$Z = 10$



Entering variable: x_2
Leaving variable: x_3

$$x_2 = 2 - 3x_3 + 2x_4$$
$$x_1 = \frac{5}{3} - \frac{1}{3}(2 - 3x_3 + 2x_4) - \frac{1}{3}x_4 = 1 + x_3 - x_4$$

$$Z = 10 + 3(2 - 3x_3 + 2x_4) - 2x_4 = 16 - 9x_3 + 4x_4$$

$$Z = 16 - 9x_3 + 4x_4$$
$$x_2 = 2 - 3x_3 + 2x_4$$
$$x_1 = 1 + x_3 - x_4$$

Basic variables: $x_1 = 1, x_2 = 2$
Non-basic variables: $x_3 = x_4 = 0$
$Z = 16$

Iteration

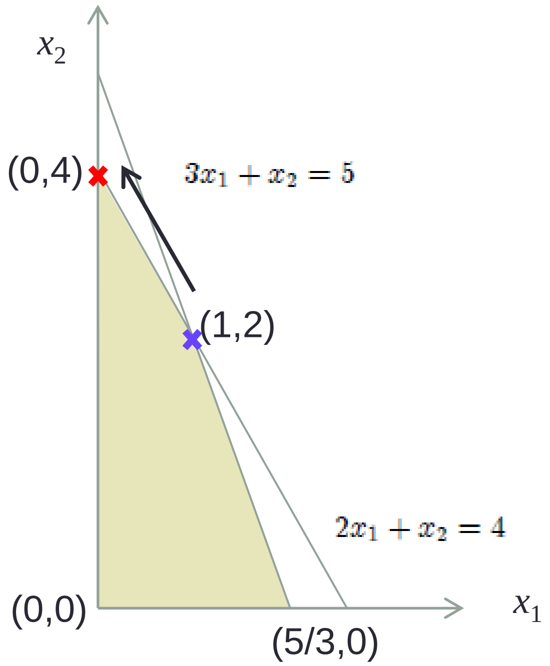
Iteration 3:

$$Z = 16 - 9x_3 + 4x_4$$
$$x_2 = 2 - 3x_3 + 2x_4$$
$$x_1 = 1 + x_3 - x_4$$

Basic variables: $x_1 = 1, x_2 = 2$
Non-basic variables: $x_3 = x_4 = 0$
$Z = 16$

Entering variable: x_4
Leaving variable: x_1

$$x_4 = 1 + x_3 - x_1$$
$$x_2 = 2 - 3x_3 + 2(1 + x_3 - x_1)$$
$$= 4 - x_3 - 2x_1$$



$$Z = 16 - 9x_3 + 4(1 + x_3 - x_1)$$
$$= 20 - 5x_3 - 4x_1$$

$$Z = 20 - 5x_3 - 4x_1$$
$$x_2 = 4 - x_3 - 2x_1$$
$$x_4 = 1 + x_3 - x_1$$

Basic variables: $x_2 = 4, x_4 = 1$
Non-basic variables: $x_1 = x_3 = 0$
$Z = 20$

SIMPLEX METHOD: TABULAR FORM

… where we see the tabular form of the simplex method which

- Is convenient to perform calculations on paper
- Records only the essential information
- Saves on a lot of rewriting

Simplex Method (Tabular Form): Initialization

Original LP

$$\max Z = 3x_1 + 5x_2$$
$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1, x_2 \geq 0$$

Augmented LP

$$\max Z = 3x_1 + 5x_2$$
$$x_1 + x_3 = 4$$
$$2x_2 + x_4 = 12$$
$$3x_1 + 2x_2 + x_5 = 18$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Initialization

$$Z = 3x_1 + 5x_2$$
$$x_3 = 4 - x_1$$
$$x_4 = 12 - 2x_2$$
$$x_5 = 18 - 3x_1 - 2x_2$$

$$Z - 3x_1 - 5x_2 = 0$$
$$x_1 + x_3 = 4$$
$$2x_2 + x_4 = 12$$
$$3x_1 + 2x_2 + x_5 = 18$$

Basic variables: $x_3 = 4, x_4 = 12, x_5 = 18$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Simplex Method (Tabular Form): Initialization

Original LP

$$\max Z = 3x_1 + 5x_2$$
$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 \le 18$$
$$x_1, x_2 \ge 0$$

Augmented LP

$$\max Z = 3x_1 + 5x_2$$
$$x_1 + x_3 = 4$$
$$2x_2 + x_4 = 12$$
$$3x_1 + 2x_2 + x_5 = 18$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Initialization

$$Z = 3x_1 + 5x_2$$
$$x_3 = 4 - x_1$$
$$x_4 = 12 - 2x_2$$
$$x_5 = 18 - 3x_1 - 2x_2$$

$$Z - 3x_1 - 5x_2 = 0$$
$$x_1 + x_3 = 4$$
$$2x_2 + x_4 = 12$$
$$3x_1 + 2x_2 + x_5 = 18$$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	-3	-5	0	0	0	0
x_3	0	1	0	1	0	0	4
x_4	0	0	2	0	1	0	12
x_5	0	3	2	0	0	1	18

Basic variables: $x_3 = 4, x_4 = 12, x_5 = 18$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration 1:

$$Z = 3x_1 + 5x_2$$
$$x_3 = 4 - x_1$$
$$x_4 = 12 - 2x_2$$
$$x_5 = 18 - 3x_1 - 2x_2$$

Basic variables: $x_3 = 4, x_4 = 12, x_5 = 18$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	-5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	$\frac{12}{2} = 6$
x_5	0	3	2	0	0	1	18	$\frac{18}{2} = 9$

1. Select entering variable: the variable with the largest positive coefficient in the obj. function
2. Select leaving variable **(min-ratio test)**:
   Choices: basic variables for which the coefficient of the entering variable in the proper format is negative
   Selection: Pick the one with the smallest ratio of the RHS constant and the absolute value of the coefficient of the entering variable

Entering variable: x_2
Leaving variable: x_4

1. Select entering variable **(pivot column)**: the variable with the most **negative** entry in the Z row
2. Select leaving variable **(pivot row) by min-ratio test**:
   1. Divide each RHS entry by the pivot column entry if the pivot column entry is strictly positive
   2. Take the row with the smallest of these ratios

   The entry in the intersection of the pivot row and the pivot col is the **pivot number**

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

$$\begin{aligned} Z &= 3x_1 + 5x_2 \\ x_3 &= 4 - x_1 \\ x_4 &= 12 - 2x_2 \\ x_5 &= 18 - 3x_1 - 2x_2 \end{aligned}$$

Basic variables: $x_3 = 4, x_4 = 12, x_5 = 18$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Entering variable: x_2
Leaving variable: x_4

3. express the entering variable as a function of leaving and non-basic variables
4. use the expression obtained at step 3 to replace the entering basic variable in the objective function and in all constraints

$$x_3 = 4 - x_1$$

$$x_2 = 6 - \frac{1}{2}x_4$$

$$x_5 = 18 - 3x_1 - 2\left(6 - \frac{1}{2}x_4\right) = 6 - 3x_1 + x_4$$

$$Z = 3x_1 + 5\left(6 - \frac{1}{2}x_4\right) = 30 + 3x_1 - \frac{5}{2}x_4$$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	−3	−5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z							
x_3							
x_2							
x_5							

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	−3	−5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/2$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z							
x_3							
x_2	0	0	1	0	$\frac{1}{2}$	0	$\frac{12}{2} = 6$
x_5							

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	-5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
$\text{Row}_1 - (-5) \times \text{Row}_3^{\text{new}}$	Z							
$\text{Row}_2 - 0 \times \text{Row}_3^{\text{new}}$	x_3							
$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/2$	x_2	0	0	1	0	$\frac{1}{2}$	0	$\frac{12}{2} = 6$
$\text{Row}_4 - 2 \times \text{Row}_3^{\text{new}}$	x_5							

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	−3	−5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
$\text{Row}_1 - (-5) \times \text{Row}_3^{\text{new}}$ →	Z	1	−3	0	0	$\frac{5}{2}$	0	30
$\text{Row}_2 - 0 \times \text{Row}_3^{\text{new}}$	x_3							
$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/2$	x_2	0	0	1	0	$\frac{1}{2}$	0	$\frac{12}{2} = 6$
$\text{Row}_4 - 2 \times \text{Row}_3^{\text{new}}$	x_5							

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	-5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
$\text{Row}_1 - (-5) \times \text{Row}_3^{\text{new}}$	Z	1	-3	0	0	$\frac{5}{2}$	0	30
$\text{Row}_2 - 0 \times \text{Row}_3^{\text{new}}$ →	x_3	0	1	0	1	0	0	4
$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/2$	x_2	0	0	1	0	$\frac{1}{2}$	0	$\frac{12}{2} = 6$
$\text{Row}_4 - 2 \times \text{Row}_3^{\text{new}}$	x_5							

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	-5	0	0	0	0	
x_3	0	1	0	1	0	0	4	
x_4	0	0	2	0	1	0	12	6
x_5	0	3	2	0	0	1	18	9

Operation	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
$\text{Row}_1 - (-5) \times \text{Row}_3^{\text{new}}$	Z	1	-3	0	0	$\frac{5}{2}$	0	30
$\text{Row}_2 - 0 \times \text{Row}_3^{\text{new}}$	x_3	0	1	0	1	0	0	4
$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/2$	x_2	0	0	1	0	$\frac{1}{2}$	0	$\frac{12}{2} = 6$
$\text{Row}_4 - 2 \times \text{Row}_3^{\text{new}}$ →	x_5	0	3	0	0	-1	1	6

Simplex Method (Tabular Form): End of Iteration

Iteration 1:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

      Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	-3	0	0	$\frac{5}{2}$	0	30
x_3	0	1	0	1	0	0	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6
x_5	0	3	0	0	-1	1	6

Simplex Method (Tabular Form): End of Iteration

Iteration 1:

$$Z = 3x_1 + 5x_2$$
$$x_3 = 4 - x_1$$
$$x_4 = 12 - 2x_2$$
$$x_5 = 18 - 3x_1 - 2x_2$$

Basic variables: $x_3 = 4, x_4 = 12, x_5 = 18$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

Entering variable: x_2
Leaving variable: x_4

3. express the entering variable as a function of leaving and non-basic variables
4. use the expression obtained at step 3 to replace the entering basic variable in the objective function and in all constraints.

$$Z = 30 + 3x_1 - \frac{5}{2}x_4$$
$$x_3 = 4 - x_1$$
$$x_2 = 6 - \frac{1}{2}x_4$$
$$x_5 = 6 - 3x_1 + x_4$$

Basic variables: $x_2 = 6, x_3 = 4, x_5 = 6$
Non-basic variables: $x_1 = x_4 = 0$
$Z = 30$

$$Z - 3x_1 + \frac{5}{2}x_4 = 30$$
$$x_1 + x_3 = 4$$
$$x_2 + \frac{1}{2}x_4 = 6$$
$$3x_1 - x_4 + x_5 = 6$$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	-3	0	0	$\frac{5}{2}$	0	30
x_3	0	1	0	1	0	0	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6
x_5	0	3	0	0	-1	1	6

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration 2:

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

1. Select entering variable **(pivot column)**: the variable with the most **negative** entry in the Z row
2. Select leaving variable **(pivot row) by min-ratio test**:
    1. Divide each RHS entry by the pivot column entry if the pivot column entry is strictly positive
    2. Take the row with the smallest of these ratios

    The number in the intersection of the pivot row and the pivot col is the **pivot number**

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z								
x_3								
x_2								
x_1								

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

      Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z								
x_3								
x_2								
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

$\text{Row}_4^{\text{new}} \leftarrow \text{Row}_4/3$ → x_1

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z								
x_3								
x_2								
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

$\text{Row}_1 - (-3) \times \text{Row}_4^{\text{new}}$

$\text{Row}_2 - 1 \times \text{Row}_4^{\text{new}}$

$\text{Row}_3 - 0 \times \text{Row}_4^{\text{new}}$

$\text{Row}_4^{\text{new}} \leftarrow \text{Row}_4/3$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3								
x_2								
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

$\text{Row}_1 - (-3) \times \text{Row}_4^{\text{new}}$ → Z

$\text{Row}_2 - 1 \times \text{Row}_4^{\text{new}}$

$\text{Row}_3 - 0 \times \text{Row}_4^{\text{new}}$

$\text{Row}_4^{\text{new}} \leftarrow \text{Row}_4/3$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

	Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
$\text{Row}_1 - (-3) \times \text{Row}_4^{\text{new}}$	Z	1	0	0	0	$\frac{3}{2}$	1	36	
$\text{Row}_2 - 1 \times \text{Row}_4^{\text{new}}$ →	x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
$\text{Row}_3 - 0 \times \text{Row}_4^{\text{new}}$	x_2								
$\text{Row}_4^{\text{new}} \leftarrow \text{Row}_4/3$	x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration 2:

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	-3	0	0	$\frac{5}{2}$	0	30	
x_3	0	1	0	1	0	0	4	4
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_5	0	3	0	0	-1	1	6	2

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   → Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

$\text{Row}_1 - (-3) \times \text{Row}_4^{\text{new}}$

$\text{Row}_2 - 1 \times \text{Row}_4^{\text{new}}$

$\text{Row}_3 - 0 \times \text{Row}_4^{\text{new}}$ →

$\text{Row}_4^{\text{new}} \leftarrow \text{Row}_4/3$

Simplex Method (Tabular Form): End of Iteration

Iteration 2:

3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to obtain the new pivot row

   For every other Row_i:

   Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

Simplex Method (Tabular Form): End of Iteration

Iteration 2:

$$Z = 30 + 3x_1 - \frac{5}{2}x_4$$
$$x_3 = 4 - x_1$$
$$x_2 = 6 - \frac{1}{2}x_4$$
$$x_5 = 6 - 3x_1 + x_4$$

Basic variables: $x_2 = 6, x_3 = 4, x_5 = 6$
Non-basic variables: $x_1 = x_4 = 0$
$Z = 30$

Entering variable: x_1
Leaving variable: x_5

3. express the entering variable as a function of non-basic variables
4. use the expression obtained at step 3 to replace the entering basic variable in the objective function and in all constraints.

$$Z = 36 - \frac{3}{2}x_4 - x_5$$
$$x_3 = 2 - \frac{1}{3}x_4 + \frac{1}{3}x_5$$
$$x_2 = 6 - \frac{1}{2}x_4$$
$$x_1 = 2 + \frac{1}{3}x_4 - \frac{1}{3}x_5$$

Basic variables: $x_1 = 2, x_2 = 6, x_3 = 2$
Non-basic variables: $x_4 = x_5 = 0$
$Z = 36$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

$$Z + \frac{3}{2}x_4 + x_5 = 36$$
$$x_3 + \frac{1}{3}x_4 - \frac{1}{3}x_5 = 2$$
$$x_2 + \frac{1}{2}x_4 = 6$$
$$x_1 - \frac{1}{3}x_4 + \frac{1}{3}x_5 = 2$$

Simplex Method (Tabular Form): Iteration steps 1 & 2

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

1. Select entering variable **(pivot column)**: the variable with the most **negative** entry in the Z row
2. Select leaving variable **(pivot row) by min-ratio test**:
   1. Divide each RHS entry by the pivot column entry if the pivot column entry is strictly positive
   2. Take the row with the smallest of these ratios

   The number in the intersection of the pivot row and the pivot col is the **pivot number**

Simplex Method (Tabular Form): Termination

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

Optimal solution: $x_1^* = 2, x_2^* = 6, x_3^* = 2$
$Z^* = 36$

Terminate
- if no entering variable
  - Optimal solution has been found
- if no leaving variable
  - Problem is unbounded

1. Select entering variable **(pivot column)**: the variable with the most **negative** entry in the Z row
2. Select leaving variable **(pivot row) by min-ratio test**:
   1. Divide each RHS entry by the pivot column entry if the pivot column entry is strictly positive
   2. Take the row with the smallest of these ratios

   The number in the intersection of the pivot row and the pivot col is the **pivot number**

Simplex Method (Tabular Form): Termination

$$Z = 36 - \frac{3}{2}x_4 - x_5$$
$$x_3 = 2 - \frac{1}{3}x_4 + \frac{1}{3}x_5$$
$$x_2 = 6 - \frac{1}{2}x_4$$
$$x_1 = 2 + \frac{1}{3}x_4 - \frac{1}{3}x_5$$

Optimality Test: No entering variable
Terminate!

Optimal solution: $x_1^* = 2, x_2^* = 6, x_3^* = 2$
$Z^* = 36$

Basic Var.	Z	x_1	x_2	x_3	x_4	x_5	RHS	ratio
Z	1	0	0	0	$\frac{3}{2}$	1	36	
x_3	0	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2	
x_2	0	0	1	0	$\frac{1}{2}$	0	6	
x_1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2	

Terminate
- if no entering variable
  - Optimal solution has been found
- if no leaving variable
  - Problem is unbounded

1. Select entering variable **(pivot column)**: the variable with the most **negative** entry in the Z row
2. Select leaving variable **(pivot row) by min-ratio test**:
   1. Divide each RHS entry by the pivot column entry if the pivot column entry is strictly positive
   2. Take the row with the smallest of these ratios

   The number in the intersection of the pivot row and the pivot col is the **pivot number**

Simplex Method (Tabular Form): Algorithm

Initialization

- Proceed similar to algebraic form and determine basic, non-basic variables
- Use objective in proper format to rewrite as $Z - c_1 x_1 - \cdots - c_n x_n = 0$
- Rewrite constraints to have all variables on the LHS and constant on the RHS
- Setup Simplex Tableau:
  - One col for each variable plus an additional RHS col
  - One row for each constraint plus an additional row for the constraint from the objective ($Z - c_1 x_1 - \cdots - c_n x_n = 0$)
  - Enter the coefficients of the corresponding variable in each constraint and the RHS
  - Specify the basic variables

Iteration

1. Select the **pivot col**: the variable with the most **negative** entry in the Z row
2. Select the **pivot row** by **min-ratio test**:
   1. Divide each RHS entry by the pivot col entry if the pivot col entry is strictly positive
   2. Select the row with the smallest of these ratios
      (the number at the intersection of the pivot row and the pivot col is the **pivot number**)
3. Replace the pivot row variable by the variable in the pivot col
4. Divide the pivot row by the pivot number to get the new pivot row. For every other Row_i:
   1. Multiply the new pivot row by the entry in the pivot col of Row_i and subtract from Row_i

Termination: Optimality Test

- If no pivot col can be found, then **the optimal solution has been found**
- If no pivot row be found, then **the problem is unbounded**

SIMPLEX METHOD TABULAR FORM

… another example

Simplex Method (Tabular Form): Initialization

Original LP

$$\max Z = 6x_1 + 5x_2$$
$$2x_1 + x_2 \leq 4$$
$$3x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

⇓

Augmented LP

$$\max Z = 6x_1 + 5x_2$$
$$2x_1 + x_2 + x_3 = 4$$
$$3x_1 + x_2 + x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

⇓

Initialization

$$Z = 6x_1 + 5x_2$$
$$x_3 = 4 - 2x_1 - x_2$$
$$x_4 = 5 - 3x_1 - x_2$$

Basic variables: $x_3 = 4, x_4 = 5$
Non-basic variables: $x_1 = x_2 = 0$
$Z = 0$

⇒

$$Z - 6x_1 - 5x_2 = 0$$
$$2x_1 + x_2 + x_3 = 4$$
$$3x_1 + x_2 + x_4 = 5$$

⇑

Basic Var.	Z	x_1	x_2	x_3	x_4	RHS
Z	1	-6	-5	0	0	0
x_3	0	2	1	1	0	4
x_4	0	3	1	0	1	5

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z							
	x_3							
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	

$\text{Row}_1 - (-6) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - 2 \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/3$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	-3	0	2	10	
	x_3							
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	

$\text{Row}_1 - (-6) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - 2 \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/3$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	

$\text{Row}_1 - (-6) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - 2 \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/3$

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z							
	x_2	0	0	1	3	-2	2	
	x_1							

$\text{Row}_1 - (-3) \times \text{Row}_2^{\text{new}}$

$\text{Row}_2^{\text{new}} \leftarrow \text{Row}_2 / (\frac{1}{3})$

$\text{Row}_3 - \frac{1}{3} \times \text{Row}_2^{\text{new}}$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	−6	−5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	−3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	−4	16	
	x_2	0	0	1	3	−2	2	
	x_1							

$\text{Row}_1 - (-3) \times \text{Row}_2^{\text{new}}$

$\text{Row}_2^{\text{new}} \leftarrow \text{Row}_2 / (\frac{1}{3})$

$\text{Row}_3 - \frac{1}{3} \times \text{Row}_2^{\text{new}}$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	−6	−5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	−3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	−4	16	
	x_2	0	0	1	3	−2	2	
	x_1	0	1	0	−1	1	1	

$\text{Row}_1 - (-3) \times \text{Row}_2^{\text{new}}$

$\text{Row}_2^{\text{new}} \leftarrow \text{Row}_2 \,/ (\frac{1}{3})$

$\text{Row}_3 - \frac{1}{3} \times \text{Row}_2^{\text{new}}$

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
0	Z	1	-6	-5	0	0	0	
	x_3	0	2	1	1	0	4	2
	x_4	0	3	1	0	1	5	$\frac{5}{3}$
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1
3	Z							
	x_2							
	x_4	0	1	0	-1	1	1	

$\text{Row}_1 - (-4) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - (-2) \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/1$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1
3	Z	1	4	0	5	0	20	
	x_2							
	x_4	0	1	0	-1	1	1	

$\text{Row}_1 - (-4) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - (-2) \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/1$

Simplex Method (Tabular Form): Iteration steps 3 & 4

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1
3	Z	1	4	0	5	0	20	
	→ x_2	0	2	1	1	0	4	
	x_4	0	1	0	-1	1	1	

$\text{Row}_1 - (-4) \times \text{Row}_3^{\text{new}}$

$\text{Row}_2 - (-2) \times \text{Row}_3^{\text{new}}$

$\text{Row}_3^{\text{new}} \leftarrow \text{Row}_3/1$

Simplex Method (Tabular Form): Iteration steps 1 & 2

Iteration	Basic Var.	Z	x_1	x_2	x_3	x_4	RHS	ratio
1	Z	1	0	-3	0	2	10	
	x_3	0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	2
	x_1	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	5
2	Z	1	0	0	9	-4	16	
	x_2	0	0	1	3	-2	2	
	x_1	0	1	0	-1	1	1	1
3	Z	1	4	0	5	0	20	
	x_2	0	2	1	1	0	4	
	x_4	0	1	0	-1	1	1	

Optimal solution is
$x_1^* = 0, x_2^* = 4, x_3^* = 0, x_4^* = 1$

No pivot col, so optimal solution has been found
Terminate!

SHADOW PRICES

… How valuable are the resources?

Tesla Example (from second lecture)

Tesla makes two models C_1 and C_2:

	C_1	C_2	total hours
Plant 1 (prepare Frame I)	1		4
Plant 2 (prepare Frame II)		2	12
Plant 3 (assembly)	3	2	18
Profit	3$	5$	

Question 1: What is the optimal product mix that maximizes the total profit?

Question 2: What is the minimum price that you should offer to buy the entire operation?

Question 3: What is the price that the company should be willing to pay for an extra working hour of plant 1, or 2, or 3?