Plan for today

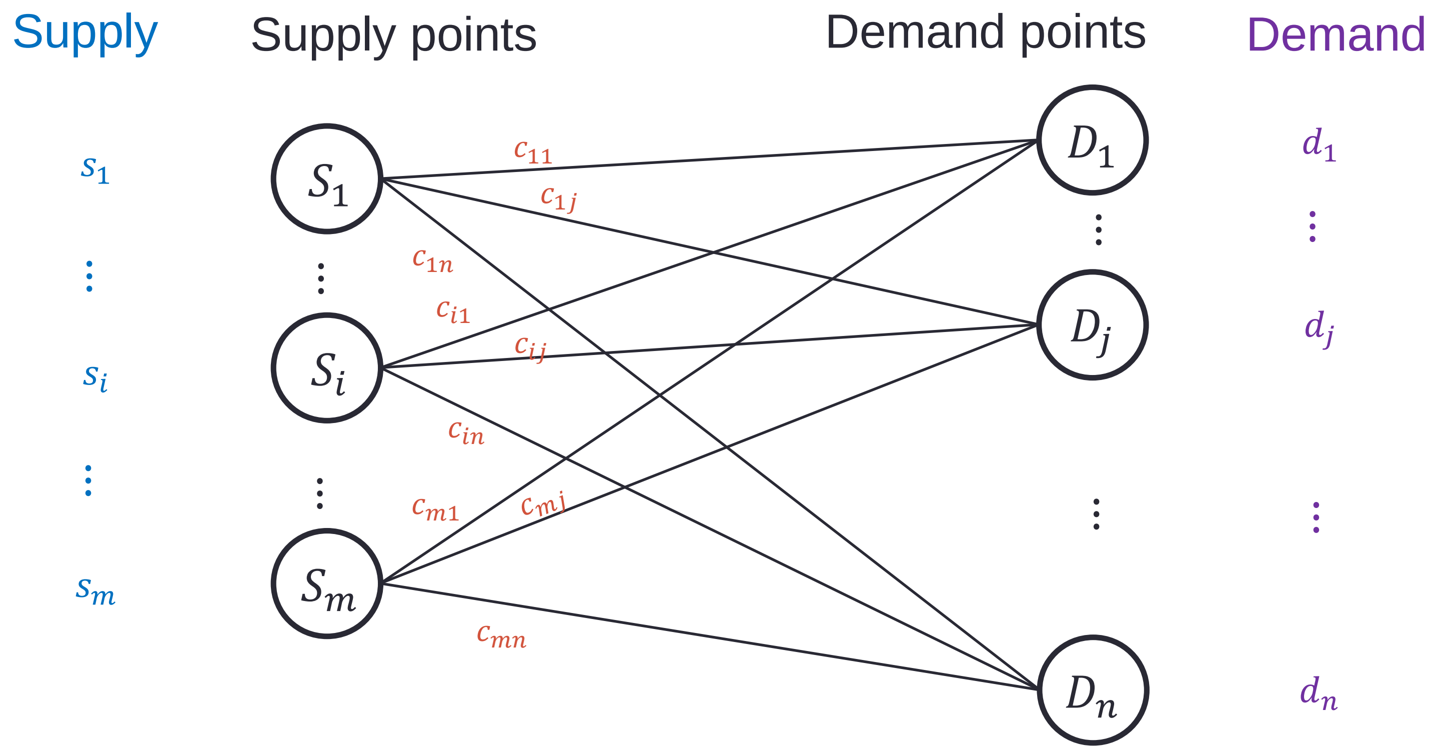
- Transportation Problem
  - Unbalanced

- Assignment Problem
  - Formulation
  - Connections to Transportation problem
  - Polygamy vs Monogamy?!
  - Solution method: Hungarian algorithm

TRANSPORTATION PROBLEM

… where we see an algorithm to solve the transportation problem

Transportation Problem: General Case



- Single product to be transported from supply to demand
- Transportation Cost per unit from S_i to D_j is $c_{ij}(\geq 0)$ for all $i \in \{1, \dots, m\}, j \in \{1, \dots, n\}$
- Supply constraints: Entire supply at every S_i must be distributed to the demand points
- Demand constraints: Entire demand at every D_j must be received from the supply points

$$\Rightarrow \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Question: How much to transport from each supply point to each demand point to minimize cost?

Formulation of the general case transportation problem

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = s_i \text{ for every } i \in \{1, \dots, m\}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \text{ for every } j \in \{1, \dots, n\}$$

$$x_{ij} \geq 0 \text{ for every } i \in \{1, \dots, m\}, j \in \{1, \dots, n\}$$

ALGORITHM FOR THE TRANSPORTATION PROBLEM

- Can use Simplex method, but it is typically slow
- So we have a special-purpose algorithm

… known as the “Transportation Simplex Method”

A Convenient Representation of the Transportation Problem

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	16	16	13	22	17	50
S_2	14	14	13	19	15	60
S_3	19	19	20	23	99	50
S_4	99	0	99	0	0	50
Demand	30	20	70	30	60	

Per unit shipment cost c_{34}

A Convenient Representation of the Transportation Problem

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	16 20	16 20	13 10	22	17	50
S_2	14 10	14	13 50	19	15	60
S_3	19	19	20 10	23 30	99 10	50
S_4	99	0	99	0	0 50	50
Demand	30	20	70	30	60	$Z = ??$

We will avoid writing the variables and
- simply write the allocations in the respective cells
- simply write the relevant allocations

Test your understanding:
What is the objective value of this solution?

Transportation Simplex Method: Two Stage Algorithm

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = s_i \text{ for every } i \in \{1, \ldots, m\}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \text{ for every } j \in \{1, \ldots, n\}$$

$$x_{ij} \geq 0 \text{ for every } i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}$$

Initialization: Identify an initial **allocation** (i.e., an initial basic feasible solution)

Allocation of values for x_{ij}

- Northwest corner rule

Iterations: Iterative method to arrive at an optimal solution

TRANSPORTATION SIMPLEX METHOD

Initialization: identifying an initial basic feasible solution

Northwest Corner Rule

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	[16] 30	[16] 20	[13]	[22]	[17]	~~50~~ 20
S_2	[14]	[14] 0	[13] 60	[19]	[15]	60
S_3	[19]	[19]	[20] 10	[23] 30	[99] 10	~~50~~ ~~40~~ 10
S_4	[99]	[0]	[99]	[0]	[0] 50	50
Demand	30	~~20~~ 0	~~70~~ 10	30	~~60~~ 50	$Z = 3460$

1. Start from northwest corner
2. Allocate as much as possible at x_{ij} while meeting demand and supply constraints
3. If allocation exhausts all supply from S_i, move one row down
4. Else if S_i has any supply remaining, move one column to the right

Basic Feasible Solution to a Transportation Problem

A **basic feasible solution** is an allocation satisfying the following conditions:

1. Supply, demand constraints and non-negativity constraints are satisfied
2. There are exactly $m + n - 1$ allocations
3. The allocations do not form a loop

the corresponding variables are **basic variables**

TRANSPORTATION SIMPLEX METHOD

Iterations: To arrive at an optimal solution

Transportation Problem Instances have Integral Opt Solutions

Observation: If the supplies and demands are integers, then the transportation problem always has an integral optimum solution.

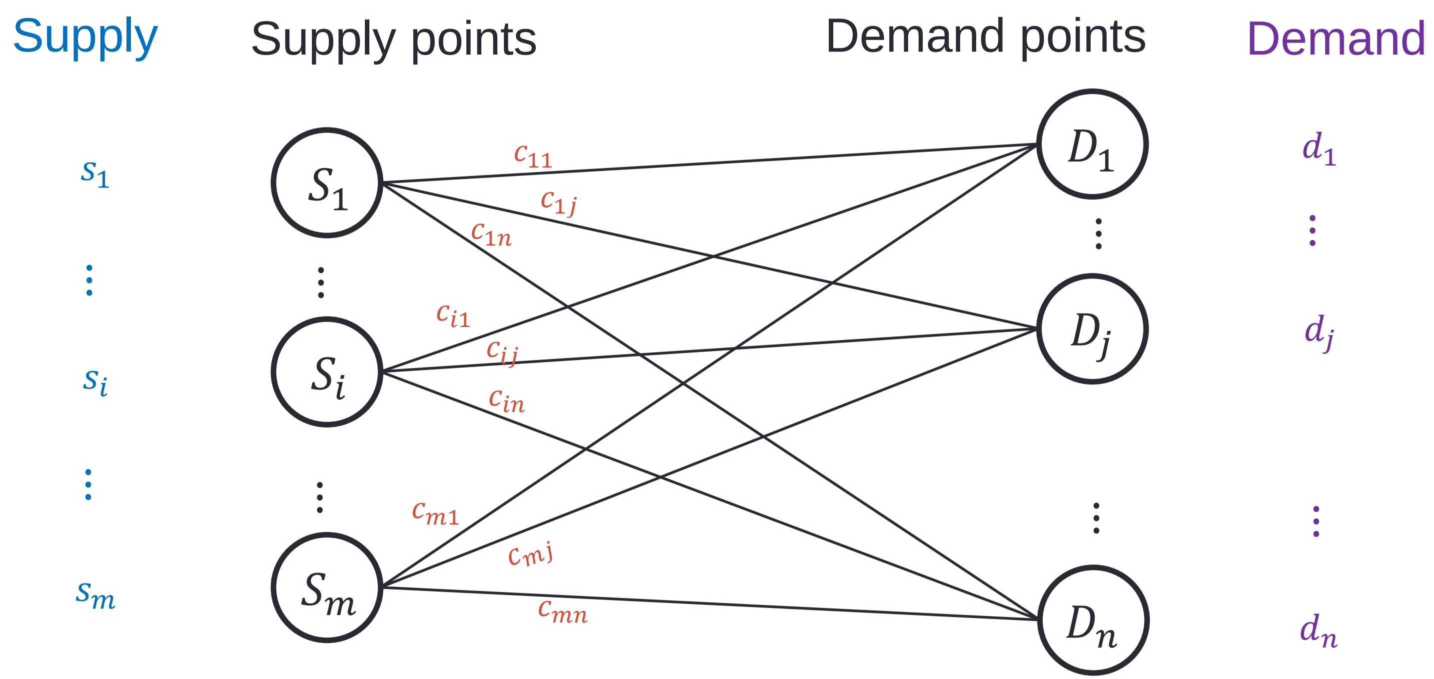
- In fact, the Transportation Simplex Method always terminates with an optimum solution whose values are integers

Proof:

- Starting allocation (initial basic feasible solution) values obtained by Northwest Corner rule are integers
- Each iteration of the transportation simplex method transforms an allocation with integer values to another allocation with integer values
- So final optimum solution values are integers

THE UNBALANCED CASE

Transportation Problem: General Case

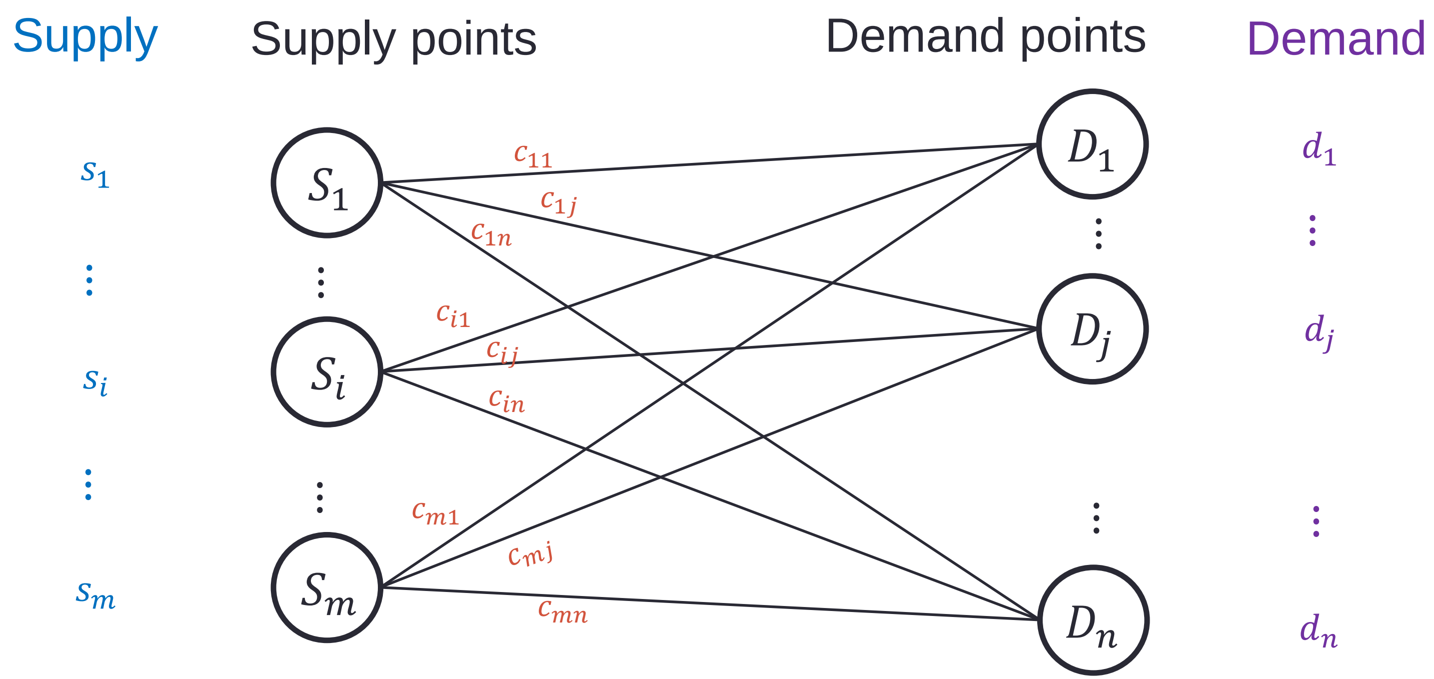


- Single product to be transported from supply to demand
- $c_{ij} \geq 0$
- Supply Constraints: S_i can supply exactly s_i units
- Demand Constraints: D_j has to receive exactly d_j units

Question: How much to transport from each supply point to each demand point to minimize cost?

Feasible only if $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

Transportation Problem: General Case



- Single product to be transported from supply to demand
- $c_{ij} \geq 0$
- Supply Constraints: S_i can supply **at most** s_i units
- Demand Constraints: D_j has to receive **at least** d_j units

Question: How much to transport from each supply point to each demand point to minimize cost?

Feasible only if $\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j$

Formulation

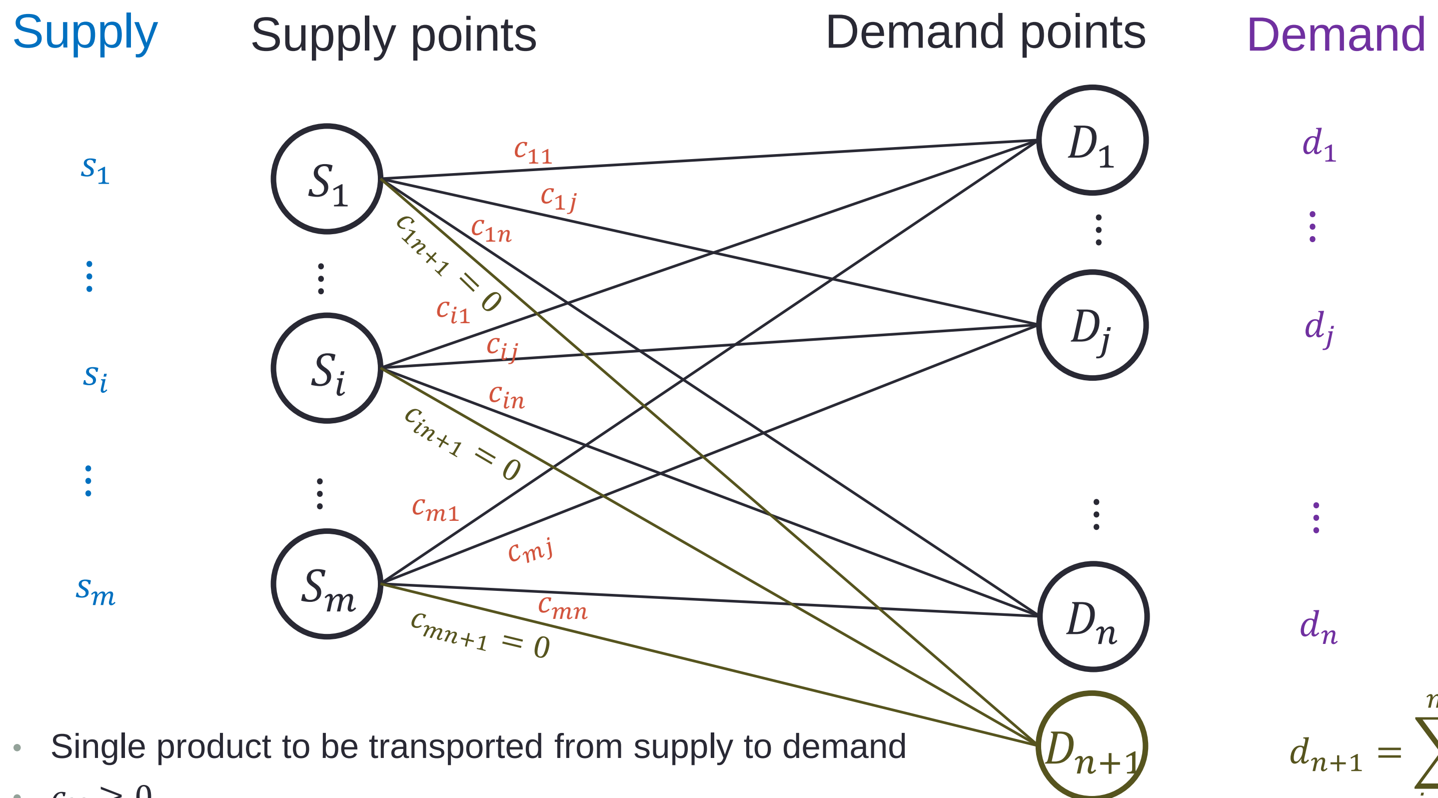
$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$
$$\sum_{j=1}^{n} x_{ij} \leq s_i \text{ for every } i \in \{1,\dots,m\}$$
$$\sum_{i=1}^{m} x_{ij} \geq d_j \text{ for every } j \in \{1,\dots,n\}$$
$$x_{ij} \geq 0 \text{ for every } i \in \{1,\dots,m\}, j \in \{1,\dots,n\}$$

=

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$
$$\sum_{j=1}^{n} x_{ij} \leq s_i \text{ for every } i \in \{1,\dots,m\}$$
$$\sum_{i=1}^{m} x_{ij} = d_j \text{ for every } j \in \{1,\dots,n\}$$
$$x_{ij} \geq 0 \text{ for every } i \in \{1,\dots,m\}, j \in \{1,\dots,n\}$$

- Since shipping costs c_{ij} are non-negative, we can assume that no demand point receives more than its required demand
- Now we have excess supply availability
- Introduce dummy demand point D_{n+1} with demand requirement $d_{n+1} = \sum_{i=1}^{m} s_i - \sum_{j}^{n} d_j$ and shipping costs $c_{i,n+1} = 0$ for every supply S_i
- Resulting transportation problem is balanced

Transportation Problem: General Case



$$d_{n+1} = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j$$

- Single product to be transported from supply to demand
- $c_{ij} \geq 0$
- Supply Constraints: S_i can supply **at most** s_i units
- Demand Constraints: D_j has to receive **at least** d_j units

Question: How much to transport from each supply point to each demand point to minimize cost?

Feasible only if $\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j$

Formulation

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n+1} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n+1} x_{ij} = s_i \text{ for every } i \in \{1, \dots, m\}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \text{ for every } j \in \{1, \dots, n+1\}$$

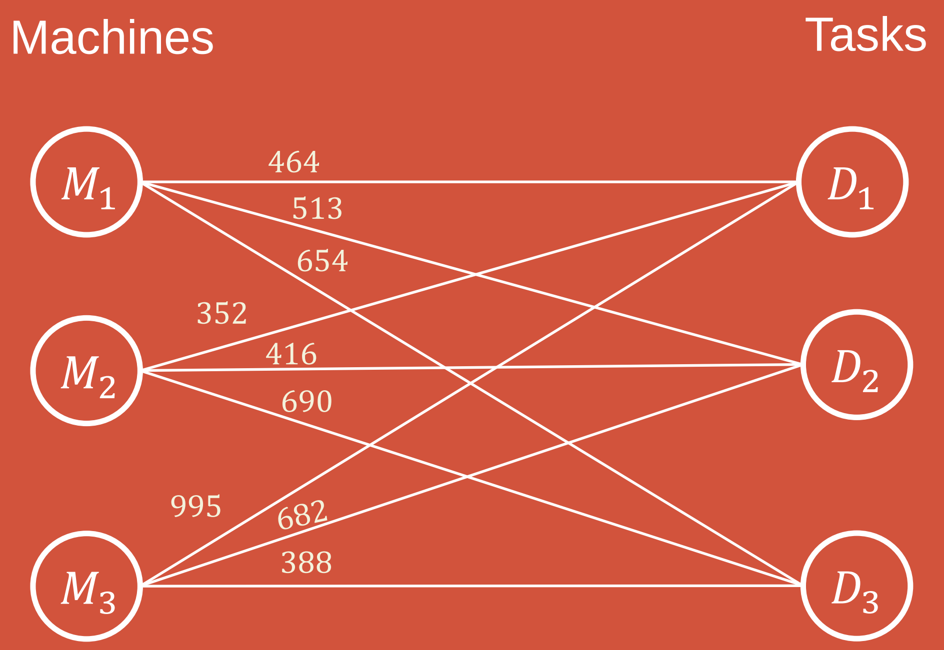
$$x_{ij} \geq 0 \text{ for every } i \in \{1, \dots, m\}, j \in \{1, \dots, n\}$$

where

$d_{n+1} := \sum_{i=1}^{m} s_i - \sum_{j}^{n} d_j$ and

$c_{i,n+1} := 0$ for every $i = 1, \dots, m$

Solution Procedure: This is the balanced transportation problem. We know how to solve the balanced case!

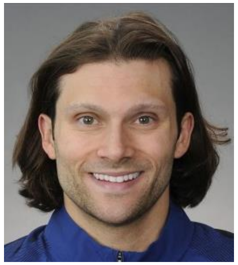


ASSIGNMENT PROBLEM

… where we see the assignment problem and an algorithm to solve it

Swimming: 4x100 relay medley

Operations … what??



	Adrian	Miller	Phelps	Murphy
Backstroke	$51.77s$	$51.99s$	$52.33s$	$51.85s$
Breaststroke	$58.86s$	$58.87s$	$58.91s$	$58.95s$
Butterfly	$51.59s$	$51.17s$	$51.14s$	$51.83s$
Freestyle	$47.85s$	$48.93s$	$48.01s$	$49.31s$

Question: Who swims which leg of the relay to minimize total time?

Example

- 4 machines and 4 tasks
- Cost for machine i to process task j is c_{ij} as given below
- Each task to be assigned to exactly one machine
- Each machine can process exactly one task

Locations
Jobs
Tasks

Franchisees / Employees / Machines

	T_1	T_2	T_3	T_4
M_1	5	9	3	6
M_2	8	7	8	2
M_3	6	10	12	7
M_4	3	10	8	6

Question: Which task should be assigned to which machine so that the total processing cost is minimized?

	T_1	T_2	T_3	T_4
M_1	5	9	3	6
M_2	8	7	8	2
M_3	6	10	12	7
M_4	3	10	8	6

Formulation

Step 1: identify decision variables

For $i \in \{1,2,3,4\}, j \in \{1,2,3,4\}$, let $x_{ij} = \begin{cases} 1 \text{ if task j is assigned to machine i} \\ 0 \text{ otherwise} \end{cases}$

Step 2: determine the objective function

$$\begin{aligned} \min Z = {} & 5\,x_{11} + 9\,x_{12} + 3\,x_{13} + 6\,x_{14} \\ & + 8\,x_{21} + 7\,x_{22} + 8\,x_{23} + 2\,x_{24} \\ & + 6\,x_{31} + 10\,x_{32} + 12\,x_{33} + 7\,x_{34} \\ & + 3\,x_{41} + 10\,x_{42} + 8\,x_{43} + 6\,x_{44} \end{aligned}$$

Step 3: identify constraints

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 1 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 1 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 1 \\ x_{41} + x_{42} + x_{43} + x_{44} &= 1 \end{aligned}\right\}$$

Machine constraints: Each machine is assigned exactly one task

$$\left.\begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} &= 1 \\ x_{12} + x_{22} + x_{32} + x_{42} &= 1 \\ x_{13} + x_{23} + x_{33} + x_{43} &= 1 \\ x_{14} + x_{24} + x_{34} + x_{44} &= 1 \end{aligned}\right\}$$

Task constraints: Each task is assigned to exactly one machine

$$x_{ij} \in \{0,1\}, i = 1,2,3,4, j = 1,2,3,4$$

Formulation

$$\min Z = 5\,x_{11} + 9\,x_{12} + 3\,x_{13} + 6\,x_{14}$$
$$+8\,x_{21} + 7\,x_{22} + 8\,x_{23} + 2\,x_{24}$$
$$+6\,x_{31} + 10\,x_{32} + 12\,x_{33} + 7\,x_{34}$$
$$+3\,x_{41} + 10\,x_{42} + 8\,x_{43} + 6\,x_{44}$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$
$$x_{ij} \in \{0,1\}, i = 1,2,3,4, j = 1,2,3,4$$

Note: Every feasible solution has exactly 4 variables that take value one, i.e., non-zero

	T_1	T_2	T_3	T_4
M_1	5	9	3	6
M_2	8	7	8	2
M_3	6	10	12	7
M_4	3	10	8	6

A non-zero variable is also known as an **assignment**

Assignment Problem: General Case

- Number of machines = Number of tasks, say they are $= n$
- Given: Cost $c_{ij} (\geq 0)$ associated with machine i performing task j
- Each machine is to be assigned to exactly one task
- Each task is to be performed by exactly one machine

	T_1	…	T_j	…	T_n
M_1	c_{11}	…	c_{1j}	…	c_{1n}
⋮	⋮	⋱	⋮	⋱	⋮
M_i	c_{i1}	…	c_{ij}	…	c_{in}
⋮	⋮	⋱	⋮	⋱	⋮
M_n	c_{n1}	…	c_{nj}	…	c_{nn}

Question: Which task should be assigned to which machine so that the total processing cost is minimized?

Formulation

	T_1	...	T_j	...	T_n
M_1	c_{11}	...	c_{1j}	...	c_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
M_i	c_{i1}	...	c_{ij}	...	c_{in}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
M_n	c_{n1}	...	c_{nj}	...	c_{nn}

Step 1: identify decision variables

For $i \in \{1, \dots, n\}, j \in \{1, \dots, n\}$, let $x_{ij} = \begin{cases} 1 \text{ if task j is assigned to machine i} \\ 0 \text{ otherwise} \end{cases}$

Step 2: determine the objective function

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Step 3: identify constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1, \dots, n\}$$

Machine constraints: Each machine is assigned exactly one task

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1, \dots, n\}$$

Task constraints: Each task is assigned to exactly one machine

$x_{ij} \in \{0,1\}$ for every $i \in \{1, \dots, n\}, j \in \{1, \dots, n\}$

Formulation of the general case assignment problem

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1, \ldots, n\}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1, \ldots, n\}$$

$$x_{ij} \in \{0,1\} \text{ for every } i \in \{1, \ldots, n\}, j \in \{1, \ldots, n\}$$

Note: Every feasible solution has exactly n non-zero variables

A non-zero variable is also known as an **assignment**

Assignment problem and Transportation problem

Problem 1

$$\min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1,\ldots,n\}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1,\ldots,n\}$$

$$x_{ij} \in \{0,1\} \text{ for every } i \in \{1,\ldots,n\}, j \in \{1,\ldots,n\}$$

=

Problem 2

$$\min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1,\ldots,n\}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1,\ldots,n\}$$

$$x_{ij} \geq 0 \text{ for every } i \in \{1,\ldots,n\}, j \in \{1,\ldots,n\}$$

This problem has an integral optimum solution

- A special case of the transportation problem
    - where $m = n$, with all demands and all supplies being 1
- Recall: "Integral Optimum Solution Property" of the transportation problem
    - So opt solution to Problem 2 will also give an opt solution to Problem 1

Assignment problem and Transportation problem

Problem 1		Problem 2
$\min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ $\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1,\dots,n\}$ $\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1,\dots,n\}$ $x_{ij} \in \{0,1\} \text{ for every } i \in \{1,\dots,n\}, j \in \{1,\dots,n\}$	=	$\min Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ $\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1,\dots,n\}$ $\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1,\dots,n\}$ $x_{ij} \geq 0 \text{ for every } i \in \{1,\dots,n\}, j \in \{1,\dots,n\}$

- Problem 2 is a “highly” degenerate transportation problem
  - n^2 variables, $2n$ constraints

    ⇒ any basic feasible solution has $2n-1$ basic variables
  - But any feasible solution has only n non-zero variables

    ⇒ Among the $2n-1$ basic variables, n basic variables take a value of one and the remaining $n-1$ basic variables take a value of zero
- Degenerate problems have too many intermediate iterations which do not improve on the objective
- So we do not use the transportation simplex method directly, but use a special-purpose algorithm

THE UNBALANCED CASE

… can be converted to the balanced case

Machine Location Problem

- 4 possible locations, 3 machines
- Cost c_{ij} for installing machine M_i in location T_j
- Each machine is to be installed in exactly one location
- Each location can accommodate at most one machine

Question: How to pick 3 locations for the 3 machines to minimize total cost?

	T_1	T_2	T_3	T_4
M_1	13	16	12	11
M_2	15	–	13	20
M_3	5	7	10	6

Location T_2 is not suitable for machine M_2

Machine Location Problem

- 4 possible locations, 3 machines
- Cost c_{ij} for installing machine M_i in location T_j
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Question: How to pick 3 locations for the 3 machines to minimize total cost?

	T_1	T_2	T_3	T_4
M_1	13	16	12	11
M_2	15	M	13	20
M_3	5	7	10	6

Location T_2 is not suitable for machine M_2

Formulation as an assignment problem:

1. Introduce big M (recall: big M = large value) for the cost of installing M_2 at T_2

Machine Location Problem

- 4 possible locations, 3 machines
- Cost c_{ij} for installing machine M_i in location T_j
- Each machine is to be installed in exactly one location
- Each location can accommodate <u>at most</u> one machine

Question: How to pick 3 locations for the 3 machines to minimize total cost?

	T_1	T_2	T_3	T_4
M_1	13	16	12	11
M_2	15	M	13	20
M_3	5	7	10	6
M_4	0	0	0	0

Location T_2 is not suitable for machine M_2

Formulation as an assignment problem:

1. Introduce big M (recall: big M = large value) for the cost of installing M_2 at T_2
2. Introduce dummy machine M_4 to account for the extra location

Now the optimum solution for the resulting assignment problem instance with $n = 4$ gives an optimum solution to our original problem

HUNGARIAN ALGORITHM

… special purpose algorithm to solve the assignment problem

Observation 1

- Number of machines = Number of tasks, say they are $= n$
- Given: Cost $c_{ij} (\geq 0)$ associated with machine i performing task j
- Each machine is to be assigned to exactly one task
- Each task is to be performed by exactly one machine

Question: How to make n assignments to minimize total cost

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for every } i \in \{1, \dots, n\}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for every } j \in \{1, \dots, n\}$$

$$x_{ij} \in \{0,1\} \text{ for every } i \in \{1, \dots, n\}, j \in \{1, \dots, n\}$$

	T_1	T_2	T_3	T_4
M_1	2	0	1	5
M_2	2	7	0	3
M_3	0	2	5	1
M_4	0	0	0	0

Obs 1: Since all c_{ij}s are ≥ 0, if we have n assignments with $Z = 0$, then it is an optimum assignment

Observation 2

- Consider first row M_1
- This machine has to be assigned to one of the four tasks
- If all tasks uniformly decrease their cost by one dollar for M_1, then the optimum assignment will not change, but only the optimum objective value will change (decrease by one dollar)

	T_1	T_2	T_3	T_4
M_1	3	6	2	1
M_2	15	M	13	20
M_3	5	7	10	6
M_4	0	0	0	0

→

	T_1	T_2	T_3	T_4
M_1	2	5	1	0
M_2	15	M	13	20
M_3	5	7	10	6
M_4	0	0	0	0

Obs 2: If all tasks uniformly decrease their cost by one dollar for M_1, then the optimum assignment will not change

Idea

- **Obs 1:** Since all c_{ij}s are ≥ 0, if we have n assignments with $Z = 0$, then it is an optimum assignment
- **Obs 2:** If all tasks uniformly decrease their cost by one dollar for M_1, then the optimum solution will not change
- Approach: Create as many zeroes in the matrix as possible by uniformly reducing rows and columns
  - If we can find n assignments with $Z = 0$ in the reduced matrix then that assignment is optimum

Steps 1 & 2

- Step 1: For each row, subtract row min from that row
- Step 2: For each col, subtract column minimum from that col
- Reduced matrix
  - each row will have at least one zero and
  - each column will have at least one zero

Original Matrix

	T_1	T_2	T_3	T_4
M_1	13	16	12	11
M_2	15	M	13	20
M_3	5	7	10	6
M_4	0	0	0	0

→

Reduced Matrix

	T_1	T_2	T_3	T_4
M_1	2	5	1	0
M_2	2	M	0	7
M_3	0	2	5	1
M_4	0	0	0	0

Step 3

- Find a maximum assignment using zero entries in the reduced cost matrix

Choose a maximum possible number of zeroes such that each row and each col has at most one chosen zero

If we can find n assignments in the reduced cost matrix, then we have an optimum

Original Matrix

	T_1	T_2	T_3	T_4
M_1	13	16	12	11
M_2	15	M	13	20
M_3	5	7	10	6
M_4	0	0	0	0

Optimum solution is
$x_{14}^* = 1, x_{23}^* = 1, x_{31}^* = 1, x_{42}^* = 1$

Optimal cost is $11 + 13 + 5 + 0 = 29$

Reduced Matrix

	T_1	T_2	T_3	T_4
M_1	2	5	1	0
M_2	2	M	0	7
M_3	0	2	5	1
M_4	0	0	0	0

Optimum solution is
$x_{14}^* = 1, x_{23}^* = 1, x_{31}^* = 1, x_{42}^* = 1$

Optimal cost is 0

Step 3: How to look for a zero-cost assignment?

Attempt 1: Process the zeroes in arbitrary order and make an assignment when possible

A situation where arbitrary processing ordering fails

	T_1	T_2	T_3	T_4
M_1	2	5	1	(0)
M_2	2	M	(0)	7
M_3	0	2	5	1
M_4	(0)	0	0	0

Step 3: How to look for a zero-cost assignment?

<u>Another rule:</u>

Repeat until no more assignments can be made:

a. If any remaining col or row has exactly one zero, then make an assignment
b. If all rows and cols have at least 2 zeroes, then pick an arbitrary assignment

- Once an assignment is made, the other zeros in the corresponding row and column cannot be used for the assignment
  - so cross out the row and column

	T_1	T_2	T_3	T_4
M_1	2	5	1	(0)
M_2	2	M	(0)	7
M_3	(0)	2	5	1
M_4	0	(0)	0	0

Watch out: Rule may still not give a maximum assignment

→ Always inspect for a zero-cost assignment

Steps 1, 2 & 3 may not lead to optimal solution

- An example

	T_1	T_2	T_3	T_4	T_5
M_1	80	0	30	120	0
M_2	80	0	30	120	0
M_3	60	60	M	80	0
M_4	60	60	M	80	0
M_5	0	90	0	0	M

- Cannot reduce further
  … since every row and every column has at least one zero
- Rule does not give a zero-cost assignment