Plan for today

- Game Theory
  - Solution Approach:
    1. Eliminating Dominant Strategies
    2. Saddle Point
    3. Graphical Method
    4. LP Method
  - Nash Equilibrium

- LP Solving in Excel

Recap

GAME THEORY

… where we see how to compute the optimal strategy for 2-player 0-sum games

WHAT IS A GAME?

… a mathematical framework for games

Two-player, zero-sum game

- A two-player zero-sum game is specified by
- $S_1, \dots, S_m$: **strategies** for Player A,
- $T_1, \dots, T_n$: **strategies** for Player B
- **Payoff table for A:** Shows the gain for Player A for each combination of strategies for the two players

B

		T_1	$\dots$	T_j	$\dots$	T_n
	S_1	p_{11}	$\dots$	p_{1j}	$\dots$	p_{1n}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
A	S_i	p_{i1}	$\dots$	p_{ij}	$\dots$	p_{in}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	S_m	p_{m1}	$\dots$	p_{mj}	$\dots$	p_{mn}

Payoff table for Player A

What would we like to understand?

- Given: payoff table for A
- Assuming that players are intelligent and rational

Question:

- With what probability (i.e., proportion) x_i should player A play each strategy S_i and
- With what probability (i.e., proportion) y_j should player B play each strategy T_j

so that A maximizes her profit and B minimizes his loss

					B		
			y_1	$\cdots$	y_j	$\cdots$	y_n
			T_1	$\cdots$	T_j	$\cdots$	T_n
	x_1	S_1	p_{11}	$\cdots$	p_{1j}	$\cdots$	p_{1n}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
A	x_i	S_i	p_{i1}	$\cdots$	p_{ij}	$\cdots$	p_{in}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
	x_m	S_m	p_{m1}	$\cdots$	p_{mj}	$\cdots$	p_{mn}

Payoff table for Player A

- Player A knows that Player B is an intelligent player and so will not allow Player A to get more and more profit
- So Player A's objective will be to maximize the minimum profit that she can get
  - Player A: **Maximin criterion**
- Similarly, Player B's objective will be to minimize the maximum loss
  - Player B: **Minimax criterion**
- **Value of the game** = Payoff to player A when both players play optimally

SOLUTION APPROACH 1

Pure Optimal Strategy: Eliminating Dominated Strategies

B

		$\boldsymbol{T_1}$	$\boldsymbol{T_2}$
A	S_1	1	2
	S_2	1	-1

Example 1

B

		T_1	T_2	T_3
A	S_1	1	2	4
	S_2	1	0	5
	~~S_3~~	~~0~~	~~1~~	~~-1~~

- Player A will never play strategy S_3
  - Since no matter what Player B plays, Player A can play strategy S_1 to gain more money
  - So, S_3 is dominated by S_1 and hence can be eliminated

Example 1

		B		
		T_1	T_2	~~T_3~~
A	S_1	1	2	~~4~~
	S_2	1	0	~~5~~
	~~S_3~~	~~0~~	~~1~~	~~-1~~

- Player B knows that Player A is intelligent
  - And so would have eliminated S_3 from consideration
- Now, for Player B, strategy T_3 is dominated by T_1
  - Regardless of whether Player A plays S_1 or S_2, Player B can play strategy T_1 to lose less money

Example 1

B

		T_1	T_2	~~T_3~~
A	S_1	1	2	~~4~~
	~~S_2~~	~~1~~	~~0~~	~~5~~
	~~S_3~~	~~0~~	~~1~~	~~-1~~

- Player A knows that Player B is intelligent
  - And so would have eliminated T_3 from consideration
- Now, for Player A, strategy S_2 is dominated by S_1
  - Regardless of whether Player B plays T_1 or T_2, Player A can play strategy S_1 to gain more money

Example 1

		B		
		T_1	~~T_2~~	~~T_3~~
A	S_1	1	~~2~~	~~4~~
	~~S_2~~	~~1~~	~~0~~	~~5~~
	~~S_3~~	~~0~~	~~1~~	~~-1~~

- Player B knows that Player A is intelligent
  - And so would have eliminated S_2 from consideration
- Now, for Player B, strategy T_2 is dominated by T_1
  - Player B can play strategy T_1 to lose less money
- So Player A always plays S_1 in order to maximize her minimum profit
- While Player B always plays T_1 in order to minimize his maximum loss
- Value of the game $= 1$

Observation: Optimum is to play a single strategy throughout.
Such an optimal strategy is known as a **pure** optimal strategy

Dominated Strategies

- A strategy S_i is **dominated** by strategy S_j if S_j is at least as good as S_i regardless of what the opponent does
- A dominated strategy can be eliminated

B

A

	$\boldsymbol{T_1}$	$\cdots$	$\boldsymbol{T_j}$	$\cdots$	$\boldsymbol{T_n}$
S_1	p_{11}	$\cdots$	p_{1j}	$\cdots$	p_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_i	p_{i1}	$\cdots$	p_{ij}	$\cdots$	p_{in}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_m	p_{m1}	$\cdots$	p_{mj}	$\cdots$	p_{mn}

Payoff table for Player A

SOLUTION APPROACH 2

Pure Optimal Strategy: Identifying a Saddle Point

			B	
		$\boldsymbol{T_1}$	$\boldsymbol{T_2}$	$\boldsymbol{T_3}$
	S_1	-3	-2	4
A	S_2	2	0	2
	S_3	5	-2	-4

Example 2

B

(min profit by playing S_i)

A

	T_1	T_2	T_3	min
S_1	-3	-2	4	-3
S_2	2	0	2	0
S_3	5	-2	-4	-4

- Consider Player A
  - By playing S_1, she could gain 4 or lose 3
  - Player B is intelligent, so will protect himself from large losses
  - So he will play T_1 and ensure that player A incurs the largest loss
  - So if player A plays S_1, then the best that she will achieve is only – 3, i.e., the row-min
  - Similarly for each row

Example 2

B

(min profit by playing S_i)

A

	T_1	T_2	T_3	min
S_1	-3	-2	4	-3
S_2	2	0	2	0
S_3	5	-2	-4	-4
max	5	0	4	

(max loss by playing T_i)

- Consider Player B
  - By playing T_1, he could lose 5 or gain 3
  - Player A is intelligent, so will protect herself from large losses
  - So she will play S_3 and ensure that player B incurs the largest loss
  - So if player B plays T_1, then the best that he will achieve is only 5, i.e., the col-max
  - Similarly for each col

Example 2

B

(min profit by playing S_i)

A

	T_1	T_2	T_3	min
S_1	-3	-2	4	-3
S_2	2	0	2	0
S_3	5	-2	-4	-4
max	5	0	4	

(max loss by playing T_i)

Player A: Maximin criterion
Maximize the minimum profit

Player B: Minimax criterion
Minimize the maximum loss

Obs. maximum among the row-min and minimum among the col-max are both achieved by the same entry

- If Player A plays a single strategy throughout, then it has to be S_2
- If Player B plays a single strategy throughout, then it has to be T_2
- If either of them deviate, then the opponent will take advantage

- So Player A always plays S_2 in order to maximize her minimum profit
- While Player B always plays T_2 in order to minimize his maximum loss
- Value of the game $= 0$

Observation: Optimum is to play a single strategy throughout. Such an optimal strategy is known as a **pure** optimal strategy

Saddle Point

- For each row consider the minimum value
- For each column consider the maximum value
- If the maximum among the row-min and
  the minimum among the col-max
  is achieved by the same entry in the payoff table
  then the entry is a **saddle point**

- The common entry in the payoff table gives the value of the game
  - The optimal strategy for Player A is the strategy corresponding to the row of this entry
  - The optimal strategy for Player B is the strategy corresponding to the col of this entry

B

A

	T_1	…	T_j	…	T_n
S_1	p_{11}	…	p_{1j}	…	p_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_i	p_{i1}	…	p_{ij}	…	p_{in}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_m	p_{m1}	…	p_{mj}	…	p_{mn}

Payoff table for Player A

Saddle point may not always exist

B

(min profit by playing S_i)

A

(max loss by playing T_i)

	T_1	T_2	min
S_1	1	-1	-1
S_2	-1	1	-1
max	1	1	

- Dominance rules do not eliminate any strategies
- No saddle point
- No pure optimum, so we need to look for **mixed** optimum

$$\begin{array}{ccc|cc} & & & \multicolumn{2}{c}{B} \\ & & & y & 1-y \\ \hline & & & T_1 & T_2 \\ \hline & x & S_1 & 1 & -1 \\ A & & & & \\ & 1-x & S_2 & -1 & 1 \end{array}$$

SOLUTION APPROACH 3

Mixed Optimum Strategy: Graphical Method

Example 3

B

			T_1	T_2	T_3
A	x	S_1	0	-2	2
	$1-x$	S_2	5	4	-3

Consider Player A

- If Player B consistently plays T_1, then Player A's gain is $0x + 5(1-x)$
- If Player B consistently plays T_2, then Player A's gain is $-2x + 4(1-x)$
- If Player B consistently plays T_3, then Player A's gain is $2x - 3(1-x)$
- Player A: Maximin criterion (Maximize the minimum profit)

$$\boxed{\max_{0\leq x\leq 1} Z = \min\{5-5x, 4-6x, -3+5x\}}$$

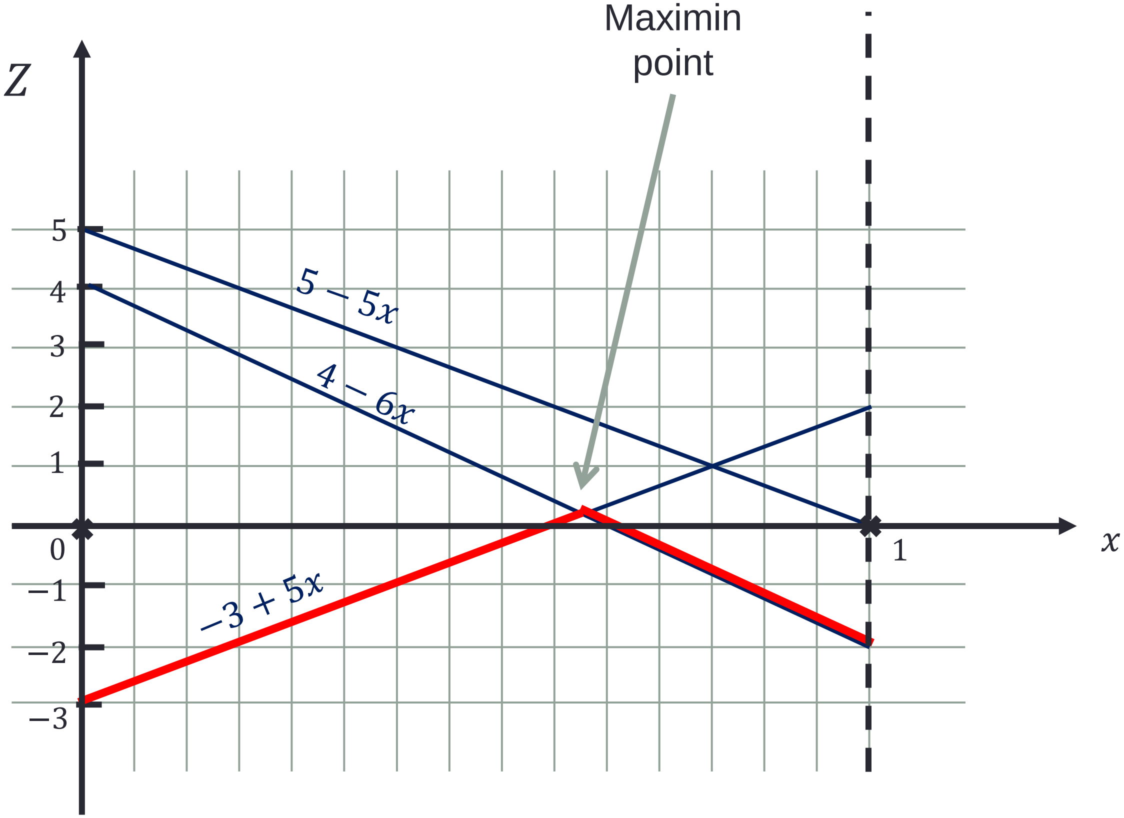
Example 3: Graphical Method

$$\max_{0\le x\le 1} Z = \min\{5-5x, 4-6x, -3+5x\}$$

B

A

		T_1	T_2	T_3
x	S_1	0	-2	2
$1-x$	S_2	5	4	-3



$$4-6x=-3+5x$$

$$x=\frac{7}{11}$$

- Optimal mixed strategy for Player A is $\left(x=\frac{7}{11}, 1-x=\frac{4}{11}\right)$
- Value of the game is $-3+5\left(\frac{7}{11}\right)=\frac{2}{11}$

Example 3

B

		y_1	y_2	$1-y_1-y_2$
		T_1	T_2	T_3
A	S_1	0	-2	2
	S_2	5	4	-3

What is the optimal strategy for Player B?

- If Player A consistently plays S_1, then Player B's loss is $0y_1 - 2y_2 + 2(1 - y_1 - y_2)$
- If Player A consistently plays S_2, then Player B's loss is $5y_1 + 4y_2 - 3(1 - y_1 - y_2)$
- Player B: Minimax criterion (Minimize the maximum loss)

$$\boxed{\min_{\substack{0\le y_1\le 1\\ 0\le y_2\le 1\\ y_1+y_2\le 1}} \; w = \max\{2 - 2y_1 - 4y_2, -3 + 2y_1 + y_2\}}$$

Two variables y_1, y_2! Cannot use graphical method!

What do we do? Next approach…

B

		y_1	y_2	$1 - y_1 - y_2$
		$\boldsymbol{T_1}$	$\boldsymbol{T_2}$	$\boldsymbol{T_3}$
A	S_1	0	-2	2
	S_2	5	4	-3

SOLUTION APPROACH 4

Mixed Optimum Strategy: LP

Example 4

B

			T_1	T_2
	x	S_1	3	-2
A	$1-x$	S_2	1	2

- Consider Player A
  - If Player B consistently plays T_1, then Player A's gain is $3x+(1-x)$
  - If Player B consistently plays T_2, then Player A's gain is $-2x+2(1-x)$
  - Player A: Maximin criterion (Maximize the minimum profit)

$$\boxed{\max_{0\le x\le 1} \ Z = \min\{2x+1, -4x+2\}} \quad = \quad \boxed{\begin{array}{c} \max u \\ u \le 2x+1 \\ u \le -4x+2 \\ 0 \le x \le 1 \end{array}}$$

which is a LP

Solving gives $x=\frac{1}{6}$

Value of the game = Player A's payoff = $\frac{4}{3}$

Example 4

B

		y	$1-y$
		T_1	T_2
A	S_1	3	-2
	S_2	1	2

- Consider Player B
  - If Player A consistently plays S_1, then Player B's loss is $3y-2(1-y)$
  - If Player A consistently plays S_2, then Player B's loss is $y+2(1-y)$
  - Player B: Minimax criterion (Minimize the maximum loss)

$$\boxed{\min_{0\le y\le 1} \; v=\max\{5y-2,-y+2\}} \quad = \quad \boxed{\begin{array}{c} \min v \\ v \ge 5y-2 \\ v \ge -y+2 \\ 0 \le y \le 1 \end{array}}$$

which is a LP

Which can be solved using
the simplex method

Back to Example 3

LP Formulation

$$\begin{array}{cccccc} & & & \multicolumn{3}{c}{B} \\ & & & y_1 & y_2 & y_3 \\ & & & T_1 & T_2 & T_3 \\ A & x_1 & S_1 & 0 & -2 & 2 \\ & x_2 & S_2 & 5 & 4 & -3 \end{array}$$

Player A

$$\begin{aligned} &\max u \\ &u \leq 5x_2 \\ &u \leq -2x_1 + 4x_2 \\ &u \leq 2x_1 - 3x_2 \\ &x_1 + x_2 = 1 \\ &x_1, x_2 \geq 0 \\ &u \text{ unrestricted} \end{aligned}$$

Player A: Maximin criterion

Maximize the minimum profit

||

Player B

$$\begin{aligned} &\min v \\ &v \geq -2y_2 + 2y_3 \\ &v \geq 5y_1 + 4y_2 - 3y_3 \\ &y_1 + y_2 + y_3 = 1 \\ &y_1, y_2, y_3 \geq 0 \\ &v \text{ unrestricted} \end{aligned}$$

Player B: Minimax criterion

Minimize the maximum loss

||

Primal

$$\begin{aligned} &\max u \\ &u - 5x_2 \leq 0 && \rightarrow q_1 \\ &u + 2x_1 - 4x_2 \leq 0 && \rightarrow q_2 \\ &u - 2x_1 + 3x_2 \leq 0 && \rightarrow q_3 \\ &x_1 + x_2 = 1 && \rightarrow w \\ &x_1, x_2 \geq 0 \\ &u \text{ unrestricted} \end{aligned}$$

Dual

$$\begin{aligned} &\min w \\ &w + 2q_2 - 2q_3 \geq 0 \\ &w - 5q_1 - 4q_2 + 3q_3 \geq 0 \\ &q_1 + q_2 + q_3 = 1 \\ &q_1, q_2, q_3 \geq 0 \\ &w \text{ unrestricted} \end{aligned}$$

Solving for Player B's opt mixed strategy

Primal

$$\begin{array}{ll} \max u & \\ u - 5x_2 \leq 0 & \rightarrow y_1 \\ u + 2x_1 - 4x_2 \leq 0 & \rightarrow y_2 \\ u - 2x_1 + 3x_2 \leq 0 & \rightarrow y_3 \\ x_1 + x_2 = 1 & \rightarrow v \\ x_1, x_2 \geq 0 & \\ u \text{ unrestricted} & \end{array}$$

$$x_1^* = \frac{7}{11}, x_2^* = \frac{4}{11}, u^* = \frac{2}{11}$$

Dual

$$\begin{array}{ll} \min v & \\ v + 2y_2 - 2y_3 \geq 0 & \rightarrow x_1 \\ v - 5y_1 - 4y_2 + 3y_3 \geq 0 & \rightarrow x_2 \\ y_1 + y_2 + y_3 = 1 & \rightarrow u \\ y_1, y_2, y_3 \geq 0 & \\ v \text{ unrestricted} & \end{array}$$

$$y_1^* = 0, y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11}, v^* = \frac{2}{11}$$

Optimal mixed strategy for Player B is $\left(y_1^* = 0, y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11}\right)$

- Strong Duality implies $v^* = \frac{2}{11}$
- Complementary slackness conditions:

$$\begin{array}{rl} y_1^*(u^* - 5x_2^*) = 0 & \Rightarrow y_1^*(-18/11) = 0 \Rightarrow y_1^* = 0 \\ y_2^*(u^* + 2x_1^* - 4x_2^*) = 0 & \Rightarrow 0y_2^* = 0 \\ y_3^*(u^* - 2x_1^* + 3x_2^*) = 0 & \Rightarrow 0y_3^* = 0 \\ x_1^*(v^* + 2y_2^* - 2y_3^*) = 0 & \left.\begin{array}{l} \Rightarrow 2y_2^* - 2y_3^* = -2/11 \\ \Rightarrow -4y_2^* + 3y_3^* = -2/11 \\ \quad y_1^* + y_2^* + y_3^* = 1 \end{array}\right\} \Rightarrow y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11} \\ x_2^*(v^* - 5y_1^* - 4y_2^* + 3y_3^*) = 0 & \end{array}$$

LP Formulation: General Case

					B		
			y_1	…	y_j	…	y_n
			T_1	…	T_j	…	T_n
	x_1	S_1	p_{11}	…	p_{1j}	…	p_{1n}
	⋮	⋮	⋮	⋱	⋮	⋱	⋮
A	x_i	S_i	p_{i1}	…	p_{ij}	…	p_{in}
	⋮	⋮	⋮	⋱	⋮	⋱	⋮
	x_m	S_m	p_{m1}	…	p_{mj}	…	p_{mn}

Given: Payoff table for Player A

Player A

$$\begin{aligned} &\max u \\ &u \leq \sum_{i=1}^{m} p_{ij} x_i \quad \forall\, j = 1, \dots, n \\ &\sum_{i=1}^{m} x_i = 1 \\ &x_i \geq 0 \quad \forall\, i = 1, \dots, m \\ &u \text{ unrestricted} \end{aligned}$$

Player A: Maximin criterion

Maximize the minimum profit

Player B

$$\begin{aligned} &\min v \\ &v \geq \sum_{j=1}^{n} p_{ij} y_j \quad \forall\, i = 1, \dots, m \\ &\sum_{j=1}^{n} y_j = 1 \\ &y_j \geq 0 \quad \forall\, j = 1, \dots, n \\ &v \text{ unrestricted} \end{aligned}$$

Player B: Minimax criterion

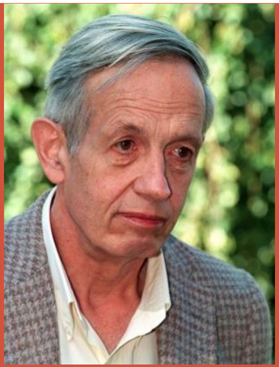
Minimize the maximum loss

A consequence of duality:

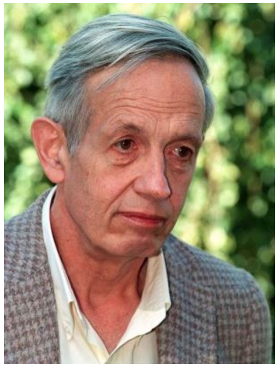
Minimax Theorem: For a pair (x^*, y^*) of mixed strategies that is optimal according to the maximin and minimax criterion,

1. the values u^* and v^* will be equal and
2. Neither player can do better by unilaterally changing her/his strategy

- i.e., player A cannot gain more than u^* by shifting to a strategy different from x^* while player B continues to play his optimal strategy y^*
- player B cannot lose less than v^* by shifting to a strategy different from y^* while player A continues to play her optimal strategy x^*



NASH EQUILIBRIUM



Equilibrium

- A pair of strategies (x^*, y^*) for players A and B is said to be a **(Nash) Equilibrium** if
  - No player can unilaterally improve her/his payoff by changing her/his strategy

I.e.,

  - Player A cannot improve her payoff by deviating from x^*
  - Equivalently, for every possible x, the payoff from (x, y^*) is not better than the payoff from (x^*, y^*)
  - Similarly for Player B

What we have seen:
Such an equilibrium exists for 2-player 0-sum games - by LP duality.

Main contribution of Nash (when he was a student):
Such an equilibrium exists for a large family of games.
(including 2-player 0-sum games)

Recall: Example 2

B

(min profit by playing S_i)

A

	T_1	T_2	T_3	min
S_1	-3	-2	4	-3
S_2	2	0	2	0
S_3	5	-2	-4	-4
max	5	0	4	

(max loss by playing T_i)

Player A: Maximin criterion
Maximize the minimum profit

Player B: Minimax criterion
Minimize the maximum loss

- If Player A plays a single strategy throughout, then it has to be S_2
- If Player B plays a single strategy throughout, then it has to be T_2
- If either of them deviate, then the opponent will take advantage

- So Player A always plays S_2 in order to maximize his minimum profit
- While Player B always plays T_2 in order to minimize his maximum loss
- Value of the game $= 0$

Here $\left(x = (0,1,0), y = (0,1,0)\right)$
is a Nash Equilibrium

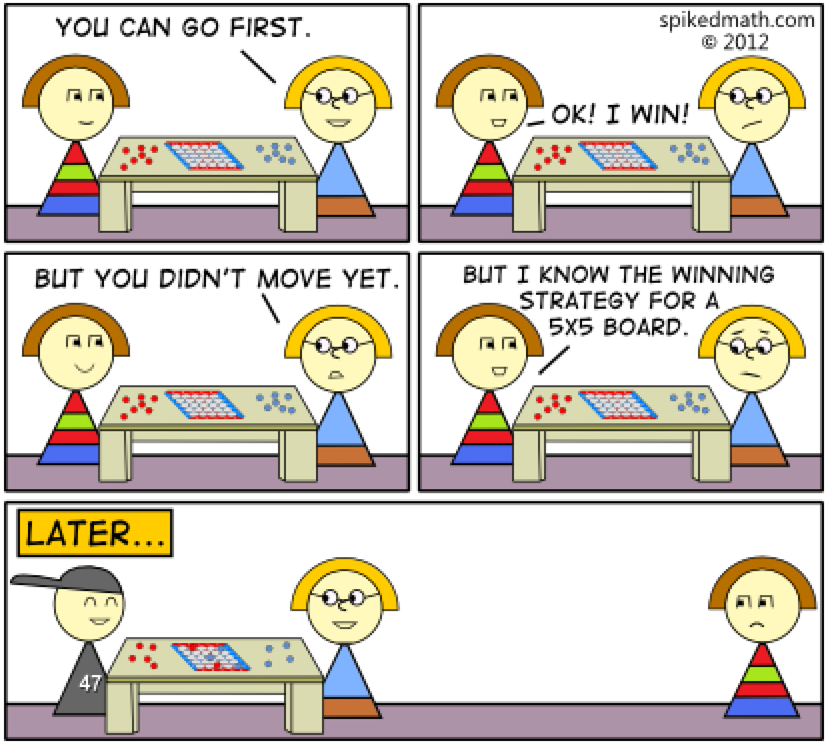
Nash Equilibrium

- A pair of strategies (x^*, y^*) for players A and B is said to be a **(Nash) Equilibrium** if
  - No player can unilaterally improve her/his payoff by changing her/his strategy

- Every pure optimal strategy is a Nash Equilibrium
- Every saddle point is a Nash Equilibrium
- Nash Equilibrium could be mixed strategies
- To verify if (x, y) is a Nash Equilibrium,
  - It is sufficient to verify if either player can improve her/his payoff when the opponent's strategy is held fixed

Be aware of the use of Game Theory

- Game Theory as a field is not simply to decide optimal strategies for games.
- As a field it provides tools that help you decide whether you should even venture into playing a game or not.
- It can help you decide whether you are going to win/lose $$$s apriori by knowing the payoff table.



LP SOLVING IN EXCEL

LP Solving in Excel

- “Solver” is an Add-In for Microsoft Excel which can solve optimization problems, including constrained problems

- Caution: It can solve only “small-sized” LP
  - E.g., at most 200 variables

Outline

- Step 0: Install Solver

- Step 1: Input Instance and Solve
    - Step 1.1: Input Data, Declare Variables, Objective, and Constraints
    - Step 1.2: Set up solver
    - Step 1.3: Solve (and get reports)

- Step 2: Interpret reports

STEP 0: INSTALL SOLVER

… To be done only if your Excel Software does not have it already installed

Step 0: Install Solver

- Windows: https://www.youtube.com/watch?v=g7C3XXyMV4A

- MacOS: https://www.youtube.com/watch?v=g7C3XXyMV4A

STEP 1: INPUT INSTANCE AND SOLVE

Example 2: Formulation

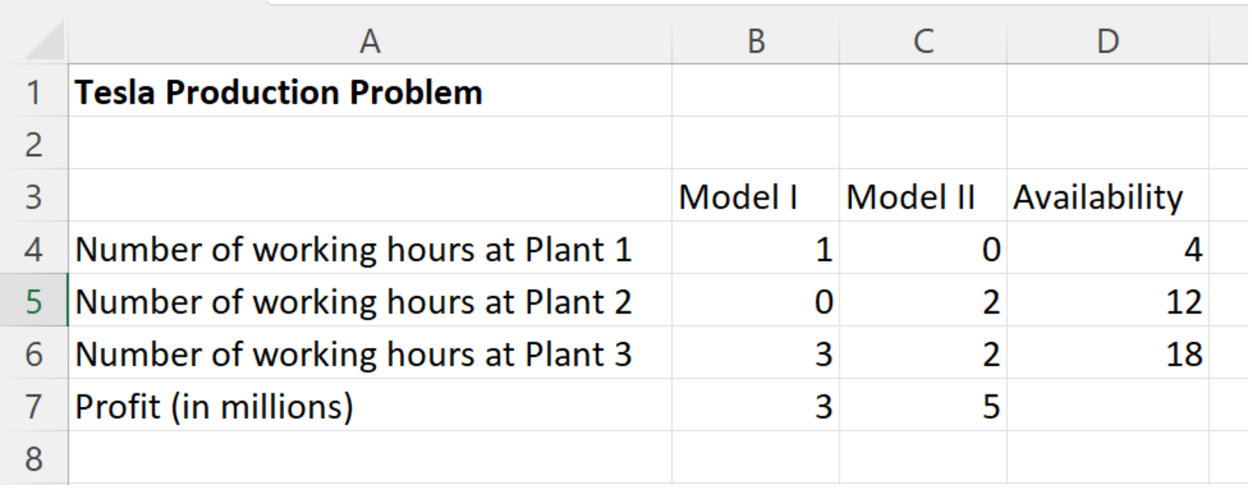
- Tesla makes two models of cars
  - Model I: makes a profit of $\$3$ million per batch
  - Model II: sells for $\$5$ million per batch
- Tesla has three plants with limited working hours
  - Plant 1: Frame I
    - at most 4 working hours per week
    - 1 hour to prepare a batch of Frame I
  - Plant 2: Frame II
    - at most 12 working hours per week
    - 2 hours to prepare a batch of Frame II
  - Plant 3: Assembly
    - at most 18 working hours per week
    - 3 hours to assemble a batch of Model I (using Frame I) and 2 hours to assemble a batch of Model II (using Frame II)

	C_1	C_2	**Availability**
P_1	1		4
P_2		2	12
P_3	3	2	18
Profit	3	5	

Question: What is the best product mix?

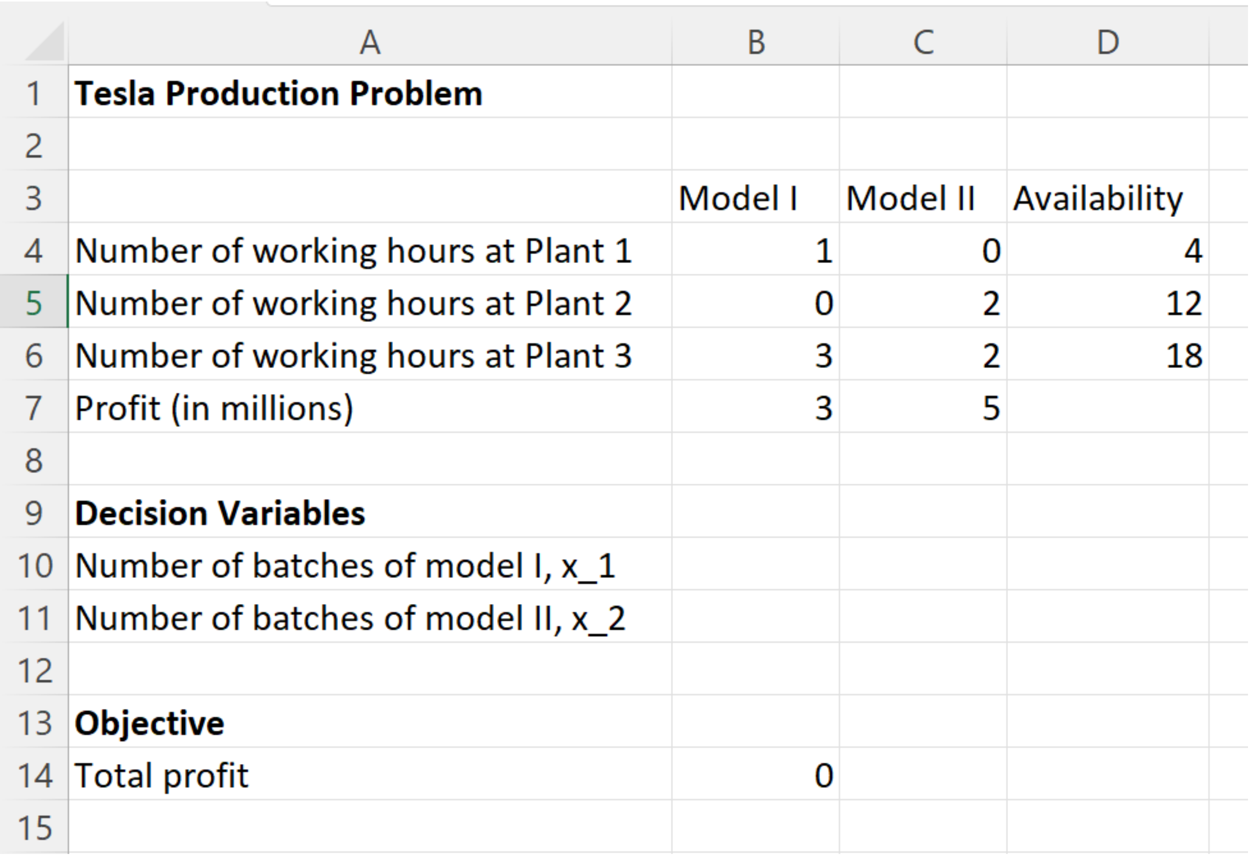
$$
\begin{aligned}
\max\ Z = 3x_1 + 5x_2 \quad & \text{(profit)} \\
x_1 \le 4 \quad & \text{(hour constraint for plant 1)} \\
2x_2 \le 12 \quad & \text{(hour constraint for plant 2)} \\
3x_1 + 2x_2 \le 18 \quad & \text{(hour constraint for plant 3)} \\
x_1 \ge 0 \quad & \text{(non-negative amount of commodity 1)} \\
x_2 \ge 0 \quad & \text{(non-negative amount of commodity 2)}
\end{aligned}
$$

Step 1.1.1: Input data



	A	B	C	D
1	**Tesla Production Problem**			
2				
3		Model I	Model II	Availability
4	Number of working hours at Plant 1	1	0	4
5	Number of working hours at Plant 2	0	2	12
6	Number of working hours at Plant 3	3	2	18
7	Profit (in millions)	3	5	
8				

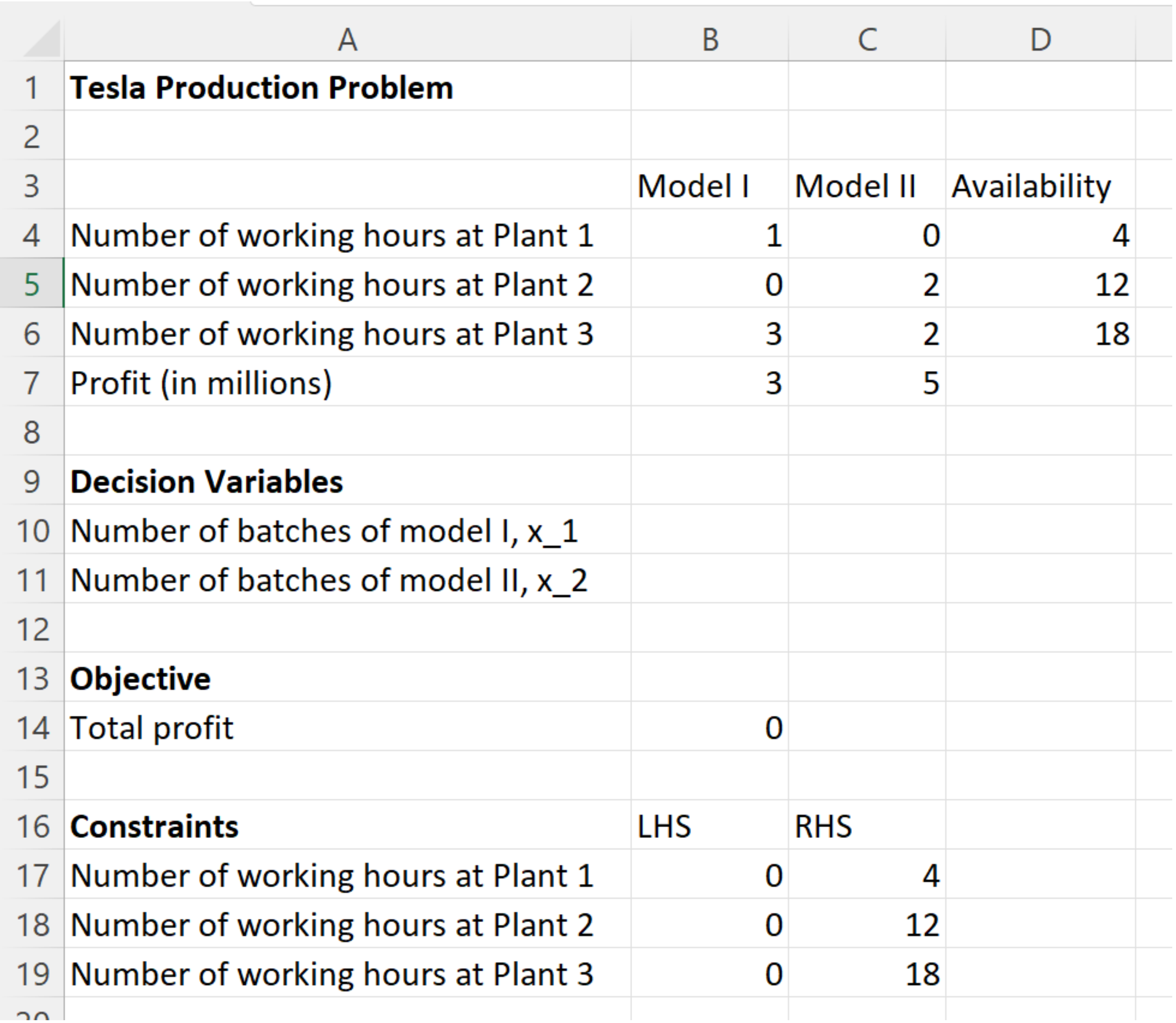
Step 1.1.2: Declare variables and objective



	A	B	C	D
1	**Tesla Production Problem**			
2				
3		Model I	Model II	Availability
4	Number of working hours at Plant 1	1	0	4
5	Number of working hours at Plant 2	0	2	12
6	Number of working hours at Plant 3	3	2	18
7	Profit (in millions)	3	5	
8				
9	**Decision Variables**			
10	Number of batches of model I, x_1			
11	Number of batches of model II, x_2			
12				
13	**Objective**			
14	Total profit	0		
15				

- Profit cell B14 is a changing cell
- It is defined as an excel function “=B7*B10+C7*B11”
  - It represents 3x_1 + 5x_2

Step 1.1.3: Declare constraints



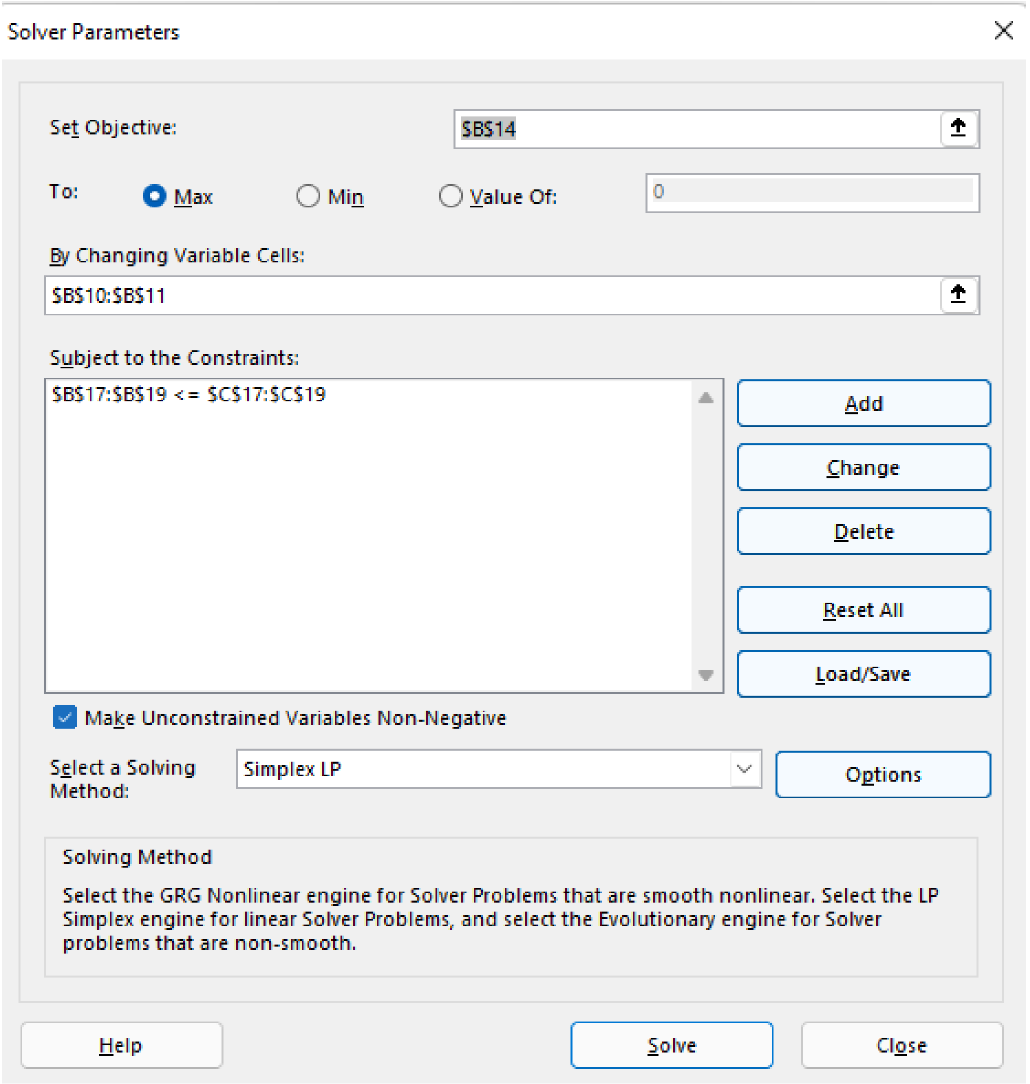
	A	B	C	D
1	**Tesla Production Problem**			
2				
3		Model I	Model II	Availability
4	Number of working hours at Plant 1	1	0	4
5	Number of working hours at Plant 2	0	2	12
6	Number of working hours at Plant 3	3	2	18
7	Profit (in millions)	3	5	
8				
9	**Decision Variables**			
10	Number of batches of model I, x_1			
11	Number of batches of model II, x_2			
12				
13	**Objective**			
14	Total profit	0		
15				
16	**Constraints**	LHS	RHS	
17	Number of working hours at Plant 1	0	4	
18	Number of working hours at Plant 2	0	12	
19	Number of working hours at Plant 3	0	18	

- LHS are changing cells: they are the LHS of constraints
- Examples:
  - B17 cell is defined as an excel function “=B4*B10+C4*B11”
    - It represents 1*x_1 + 0*x_2
  - B18 cell is defined as an excel function “=B5*B10+C5∗B11”
    - It represents 0*x_1 + 2x_2

Step 1.2: Setup Solver

- Step 2.1: Open Solver Dialog
  - Data -> Solver
- Step 2.2: Setup Solver
  - Select the cell that represents the “Objective”
    - Choose objective cell B14
  - Check “Max” to indicate maximization
  - Select the cells that represent the “Variables”
    - Choose B10:B11 (press “Shift” key to select many cells)
  - Select the cells that represent the “Constraints”
    - Constraints can only involve two adjacent columns
    - Choose B17-B19 and C17-C19
  - Check “Make Unconstrained Variables Non-Negative” (if needed)
  - Select “Simplex LP” to tell the Solver that this is an LP
  - Click ”Solve”

Step 1.2: Setup Solver



Set Objective: $B$14

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells: $B$10:$B$11

Subject to the Constraints:

$B$17:$B$19 <= $C$17:$C$19

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Tips for declaring constraints

The constraint

$B$17:$B$19 <= $C$17:$C$19

is the same as the system of constraints:

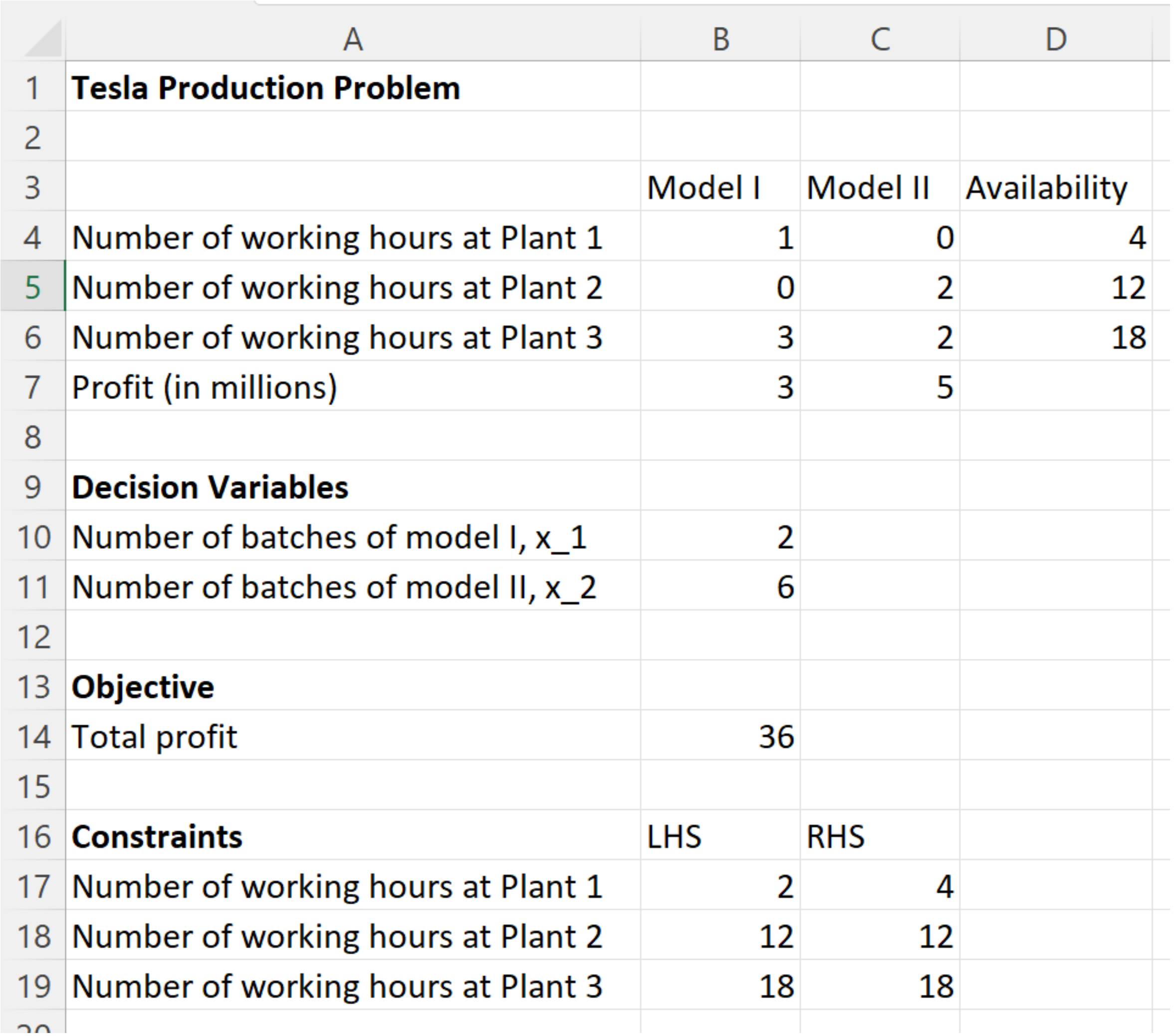
$B$17 <= $C$17

$B$18 <= $C$18

$B$19 <= $C$19

But much more convenient to use!

Step 1.3: Solve (and get reports)



	A	B	C	D
1	**Tesla Production Problem**			
2				
3		Model I	Model II	Availability
4	Number of working hours at Plant 1	1	0	4
5	Number of working hours at Plant 2	0	2	12
6	Number of working hours at Plant 3	3	2	18
7	Profit (in millions)	3	5	
8				
9	**Decision Variables**			
10	Number of batches of model I, x_1	2		
11	Number of batches of model II, x_2	6		
12				
13	**Objective**			
14	Total profit	36		
15				
16	**Constraints**	LHS	RHS	
17	Number of working hours at Plant 1	2	4	
18	Number of working hours at Plant 2	12	12	
19	Number of working hours at Plant 3	18	18	

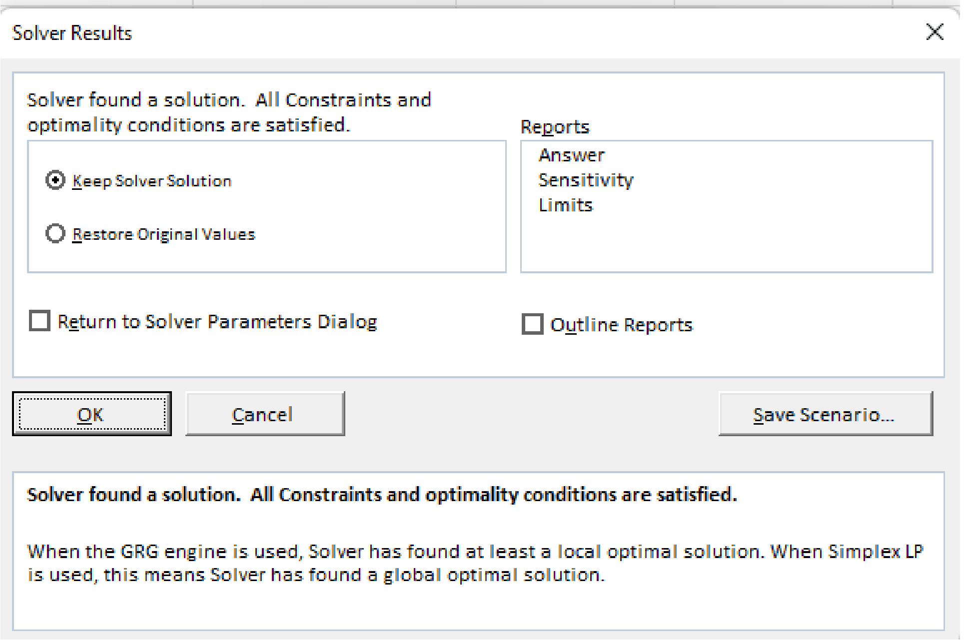
After clicking solve, see two changes:

- **Change 1:** values in "Decision Variables", "Objective", and LHS cells will change
- Optimum decision variable values: (2,6)
- Optimum objective value: 36

Step 1.3: Solve (and get reports)

After clicking solve, see two changes:

- **Change 2:** solver window becomes



Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution

○ Restore Original Values

☐ Return to Solver Parameters Dialog

☐ Outline Reports

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

- There are 3 possible reports. Each one will be a separate tab in the excel file.

STEP 2: INTERPRET REPORTS

1. Answer Report
2. Sensitivity Report
3. Limits Report: No useful information