Plan for today

• Game Theory
  • Solution Approach:
    1. Eliminating Dominant Strategies
    2. Saddle Point
    3. Graphical Method
    4. LP Method
  • Nash Equilibrium

• LP Solving in Excel
GAME THEORY

… where we see how to compute the optimal strategy for 2-player 0-sum games
WHAT IS A GAME?

... a mathematical framework for games
Two-player, zero-sum game

- A two-player zero-sum game is specified by
  - $S_1, \ldots, S_m$: strategies for Player $A$,
  - $T_1, \ldots, T_n$: strategies for Player $B$
- **Payoff table for $A$:** Shows the gain for Player $A$ for each combination of strategies for the two players

```
<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$\ldots$</th>
<th>$T_j$</th>
<th>$\ldots$</th>
<th>$T_n$</th>
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<tbody>
<tr>
<td>$S_1$</td>
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<td>$p_{1j}$</td>
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<td>$p_{1n}$</td>
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<tr>
<td>$S_i$</td>
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<td>$\ldots$</td>
<td>$p_{mj}$</td>
<td>$\ldots$</td>
<td>$p_{mn}$</td>
</tr>
</tbody>
</table>
```

Payoff table for Player $A$
What would we like to understand?

- Given: payoff table for \( A \)
- Assuming that players are intelligent and rational

**Question:**
- With what probability (i.e., proportion) \( x_i \) should player A play each strategy \( S_i \) and
- With what probability (i.e., proportion) \( y_j \) should player B play each strategy \( T_j \) so that \( A \) maximizes her profit and \( B \) minimizes his loss

- Player \( A \) knows that Player \( B \) is an intelligent player and so will not allow Player \( A \) to get more and more profit
- So Player \( A \)’s objective will be to maximize the minimum profit that she can get
  - Player \( A \): **Maximin criterion**
- Similarly, Player \( B \)’s objective will be to minimize the maximum loss
  - Player \( B \): **Minimax criterion**
- **Value of the game** = Payoff to player \( A \) when both players play optimally
SOLUTION APPROACH 1

Pure Optimal Strategy: Eliminating Dominated Strategies

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<tr>
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<th>$T_1$</th>
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</thead>
<tbody>
<tr>
<td>$S_1$</td>
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<td>2</td>
</tr>
<tr>
<td>$S_2$</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>
Example 1

- Player A will never play strategy $S_3$
  - Since no matter what Player $B$ plays, Player $A$ can play strategy $S_1$ to gain more money
  - So, $S_3$ is dominated by $S_1$ and hence can be eliminated
Example 1

- Player $B$ knows that Player $A$ is intelligent
  - And so would have eliminated $S_3$ from consideration
- Now, for Player $B$, strategy $T_3$ is dominated by $T_1$
  - Regardless of whether Player $A$ plays $S_1$ or $S_2$, Player $B$ can play strategy $T_1$ to lose less money
Example 1

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
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<td>2</td>
<td>4</td>
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<tr>
<td>$S_2$</td>
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</tr>
<tr>
<td>$S_3$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Player $A$ knows that Player $B$ is intelligent
  - And so would have eliminated $T_3$ from consideration
- Now, for Player $A$, strategy $S_2$ is dominated by $S_1$
  - Regardless of whether Player $B$ plays $T_1$ or $T_2$, Player $A$ can play strategy $S_1$ to gain more money
Example 1

- Player $B$ knows that Player $A$ is intelligent
  - And so would have eliminated $S_2$ from consideration
- Now, for Player $B$, strategy $T_2$ is dominated by $T_1$
  - Player $B$ can play strategy $T_1$ to lose less money
- So Player $A$ always plays $S_1$ in order to maximize her minimum profit
- While Player $B$ always plays $T_1$ in order to minimize his maximum loss
- Value of the game = 1

**Observation:** Optimum is to play a single strategy throughout. Such an optimal strategy is known as a **pure** optimal strategy
Dominated Strategies

- A strategy $S_i$ is **dominated** by strategy $S_j$ if $S_j$ is at least as good as $S_i$ regardless of what the opponent does.
- A dominated strategy can be eliminated.

Payoff table for Player $A$

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>...</th>
<th>$T_j$</th>
<th>...</th>
<th>$T_n$</th>
</tr>
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<tbody>
<tr>
<td>$S_1$</td>
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<td>...</td>
<td>$p_{1j}$</td>
<td>...</td>
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<td>...</td>
<td>...</td>
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<td>...</td>
<td>...</td>
</tr>
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<td>...</td>
<td>$p_{ij}$</td>
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</table>
SOLUTION APPROACH 2

Pure Optimal Strategy: Identifying a Saddle Point

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<tbody>
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<td>2</td>
</tr>
<tr>
<td>$S_3$</td>
<td>5</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>
Example 2

Consider Player $A$

- By playing $S_1$, she could gain 4 or lose 3
- Player $B$ is intelligent, so will protect himself from large losses
- So he will play $T_1$ and ensure that player $A$ incurs the largest loss
- So if player $A$ plays $S_1$, then the best that she will achieve is only $-3$, i.e., the row-min
- Similarly for each row

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>$S_2$</td>
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<td>0</td>
</tr>
<tr>
<td>$S_3$</td>
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<td>-2</td>
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Example 2

<table>
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</tr>
<tr>
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</tr>
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<td>-4</td>
</tr>
<tr>
<td>( \text{max} )</td>
<td>5</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

- **Consider Player** $B$
  - By playing $T_1$, he could lose 5 or gain 3
  - Player $A$ is intelligent, so will protect herself from large losses
  - So she will play $S_3$ and ensure that player $B$ incurs the largest loss
  - So if player $B$ plays $T_1$, then the best that he will achieve is only 5, i.e., the col-max
  - Similarly for each col
Example 2

If Player $A$ plays a single strategy throughout, then it has to be $S_2$.

If Player $B$ plays a single strategy throughout, then it has to be $T_2$.

If either of them deviate, then the opponent will take advantage.

So Player $A$ always plays $S_2$ in order to maximize her minimum profit.

While Player $B$ always plays $T_2$ in order to minimize his maximum loss.

Value of the game $= 0$.

Observation: Optimum is to play a single strategy throughout. Such an optimal strategy is known as a pure optimal strategy.
Saddle Point

• For each row consider the minimum value
• For each column consider the maximum value
• If the maximum among the row-min and the minimum among the col-max is achieved by the same entry in the payoff table then the entry is a saddle point

• The common entry in the payoff table gives the value of the game
  • The optimal strategy for Player A is the strategy corresponding to the row of this entry
  • The optimal strategy for Player B is the strategy corresponding to the col of this entry

<table>
<thead>
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<td>$S_m$</td>
<td>$p_{m1}$</td>
<td>$p_{mj}$</td>
<td>$p_{mn}$</td>
</tr>
</tbody>
</table>

Payoff table for Player $A$
Saddle point may not always exist

- Dominance rules do not eliminate any strategies
- No saddle point
- No pure optimum, so we need to look for **mixed** optimum
SOLUTION APPROACH 3

Mixed Optimum Strategy: Graphical Method

\[ T_1 x \quad T_2 x \]
\[ S_1 \quad 1 \quad -1 \]
\[ S_2 \quad -1 \quad 1 \]
Consider Player $A$

- If Player $B$ consistently plays $T_1$, then Player $A$’s gain is $0x + 5(1 - x)$
- If Player $B$ consistently plays $T_2$, then Player $A$’s gain is $-2x + 4(1 - x)$
- If Player $B$ consistently plays $T_3$, then Player $A$’s gain is $2x - 3(1 - x)$
- **Player $A$: Maximin criterion** (Maximize the minimum profit)

\[
\max_Z = \min\{5 - 5x, 4 - 6x, -3 + 5x\}
\]
Example 3: Graphical Method

\[
\max_{0 \leq x \leq 1} Z = \min \{5 - 5x, 4 - 6x, -3 + 5x\}
\]

- Optimal mixed strategy for Player A is \(x = \frac{7}{11}, 1 - x = \frac{4}{11}\)
- Value of the game is \(-3 + 5 \left(\frac{7}{11}\right) = \frac{2}{11}\)
Example 3

What is the optimal strategy for Player $B$?

- If Player $A$ consistently plays $S_1$, then Player $B$’s loss is $0y_1 - 2y_2 + 2(1 - y_1 - y_2)$
- If Player $A$ consistently plays $S_2$, then Player $B$’s loss is $5y_1 + 4y_2 - 3(1 - y_1 - y_2)$
- **Player $B$: Minimax criterion** (Minimize the maximum loss)

$$
\min_{0 \leq y_1 \leq 1} \max_{0 \leq y_2 \leq 1} \min_{y_1 + y_2 \leq 1} w = \max\{2 - 2y_1 - 4y_2, -3 + 2y_1 + y_2\}
$$

Two variables $y_1, y_2$! Cannot use graphical method! 🤔

What do we do? Next approach…
SOLUTION APPROACH 4

Mixed Optimum Strategy: LP

\[ \begin{array}{c|ccc}
   & T_1 & T_2 & T_3 \\
\hline
S_1 & 0 & -2 & 2 \\
S_2 & 5 & 4 & -3 \\
\end{array} \]
**Example 4**

<table>
<thead>
<tr>
<th></th>
<th>(T_1)</th>
<th>(T_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(S_1)</td>
<td>3</td>
</tr>
<tr>
<td>(A)</td>
<td>(S_2)</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Consider Player** \(A\)
  - If Player \(B\) consistently plays \(T_1\), then Player \(A\)'s gain is \(3x + (1 - x)\)
  - If Player \(B\) consistently plays \(T_2\), then Player \(A\)'s gain is \(-2x + 2(1 - x)\)
  - **Player \(A\): Maximin criterion** (Maximize the minimum profit)

\[
\begin{align*}
\max_{0 \leq x \leq 1} Z &= \min\{2x + 1, -4x + 2\} \\
\end{align*}
\]

\[
\begin{align*}
\max u \\
u &\leq 2x + 1 \\
u &\leq -4x + 2 \\
0 &\leq x \leq 1
\end{align*}
\]

which is a LP

Solving gives \(x = \frac{1}{6}\)

Value of the game = Player \(A\)'s payoff = \(\frac{4}{3}\)
Example 4

Consider Player $B$

- If Player $A$ consistently plays $S_1$, then Player $B$'s loss is $3y - 2(1 - y)$
- If Player $A$ consistently plays $S_2$, then Player $B$'s loss is $y + 2(1 - y)$

Player $B$: Minimax criterion (Minimize the maximum loss)

\[
\min_{0 \leq y \leq 1} v = \max \{5y - 2, -y + 2\} = \min v
\]

which is a LP

Which can be solved using the simplex method
LP Formulation

Player A: Maximin criterion
Maximize the minimum profit

\[
\begin{align*}
\text{max } u \\
u &\leq 5x_2 \\
u &\leq -2x_1 + 4x_2 \\
u &\leq 2x_1 - 3x_2 \\
x_1 + x_2 &= 1 \\
x_1, x_2 &\geq 0 \\
u &\text{ unrestricted}
\end{align*}
\]

Player B: Minimax criterion
Minimize the maximum loss

\[
\begin{align*}
\text{min } v \\
v &\geq -2y_2 + 2y_3 \\
v &\geq 5y_1 + 4y_2 - 3y_3 \\
y_1 + y_2 + y_3 &= 1 \\
y_1, y_2, y_3 &\geq 0 \\
v &\text{ unrestricted}
\end{align*}
\]
Solving for Player $B$’s opt mixed strategy

**Primal**

$$\begin{align*}
\text{max } u \\
u - 5x_2 &\leq 0 \\
u + 2x_1 - 4x_2 &\leq 0 \\
u - 2x_1 + 3x_2 &\leq 0 \\
x_1 + x_2 &= 1 \\
x_1, x_2 &\geq 0 \\
u &\text{ unrestricted}
\end{align*}$$

$$x_1^* = \frac{7}{11}, x_2^* = \frac{4}{11}, u^* = \frac{2}{11}$$

**Dual**

$$\begin{align*}
\min v \\
v + 2y_2 - 2y_3 &\geq 0 \\
v - 5y_1 - 4y_2 + 3y_3 &\geq 0 \\
y_1 + y_2 + y_3 &= 1 \\
y_1, y_2, y_3 &\geq 0 \\
v &\text{ unrestricted}
\end{align*}$$

$$y_1^* = 0, y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11}, v^* = \frac{2}{11}$$

Optimal mixed strategy for Player $B$ is $\left( y_1^* = 0, y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11} \right)$

- Strong Duality implies $v^* = \frac{2}{11}$
- Complementary slackness conditions:

$$\begin{align*}
y_1^*(u^* - 5x_2^*) &= 0 \\
y_2^*(u^* + 2x_1^* - 4x_2^*) &= 0 \\
y_3^*(u^* - 2x_1^* + 3x_2^*) &= 0 \\
x_1^*(v^* + 2y_2^* - 2y_3^*) &= 0 \\
x_2^*(v^* - 5y_1^* - 4y_2^* + 3y_3^*) &= 0
\end{align*}$$

$$\Rightarrow y_1^*(- \frac{18}{11}) = 0 \Rightarrow y_1^* = 0$$

$$\Rightarrow 0y_2^* = 0 \Rightarrow 0y_3^* = 0$$

$$\Rightarrow 2y_2^* - 2y_3^* = -\frac{2}{11} \Rightarrow -4y_2^* + 3y_3^* = -\frac{2}{11}$$

$$\Rightarrow y_1^* + y_2^* + y_3^* = 1$$

$$\Rightarrow y_2^* = \frac{5}{11}, y_3^* = \frac{6}{11}$$
LP Formulation: General Case

A consequence of duality:

**Minimax Theorem:** For a pair \((x^*, y^*)\) of mixed strategies that is optimal according to the maximin and minimax criterion,

1. the values \(u^*\) and \(v^*\) will be equal and
2. Neither player can do better by unilaterally changing her/his strategy
   - i.e., player A cannot gain more than \(u^*\) by shifting to a strategy different from \(x^*\) while player B continues to play his optimal strategy \(y^*\)
   - player B cannot lose less than \(v^*\) by shifting to a strategy different from \(y^*\) while player A continues to play her optimal strategy \(x^*\)
NASH EQUILIBRIUM
Equilibrium

- A pair of strategies \((x^*, y^*)\) for players \(A\) and \(B\) is said to be a **(Nash) Equilibrium** if
  - No player can unilaterally improve her/his payoff by changing her/his strategy
  i.e.,
  - Player \(A\) cannot improve her payoff by deviating from \(x^*\)
  - Equivalently, for every possible \(x\), the payoff from \((x, y^*)\) is not better than the payoff from \((x^*, y^*)\)
  - Similarly for Player \(B\)

What we have seen:
Such an equilibrium exists for 2-player 0-sum games - by LP duality.

Main contribution of Nash (when he was a student):
Such an equilibrium exists for a large family of games.
(including 2-player 0-sum games)
Recall: Example 2

- If Player $A$ plays a single strategy throughout, then it has to be $S_2$
- If Player $B$ plays a single strategy throughout, then it has to be $T_2$
- If either of them deviate, then the opponent will take advantage

- So Player $A$ always plays $S_2$ in order to maximize his minimum profit
- While Player $B$ always plays $T_2$ in order to minimize his maximum loss
- Value of the game = 0

Here $(x = (0,1,0), y = (0,1,0))$ is a Nash Equilibrium
Nash Equilibrium

- A pair of strategies \((x^*, y^*)\) for players \(A\) and \(B\) is said to be a (Nash) Equilibrium if
  - No player can unilaterally improve her/his payoff by changing her/his strategy

- Every pure optimal strategy is a Nash Equilibrium
- Every saddle point is a Nash Equilibrium
- Nash Equilibrium could be mixed strategies
- To verify if \((x, y)\) is a Nash Equilibrium,
  - It is sufficient to verify if either player can improve her/his payoff when the opponent’s strategy is held fixed
Be aware of the use of Game Theory

- Game Theory as a field is not simply to decide optimal strategies for games.
- As a field it provides tools that help you decide whether you should even venture into playing a game or not.
- It can help you decide whether you are going to win/lose $$$s apriori by knowing the payoff table.
LP SOLVING IN EXCEL
LP Solving in Excel

• “Solver” is an Add-In for Microsoft Excel which can solve optimization problems, including constrained problems

• Caution: It can solve only “small-sized” LP
  • E.g., at most 200 variables
Outline

- Step 0: Install Solver

- Step 1: Input Instance and Solve
  - Step 1.1: Input Data, Declare Variables, Objective, and Constraints
  - Step 1.2: Set up solver
  - Step 1.3: Solve (and get reports)

- Step 2: Interpret reports
STEP 0: INSTALL SOLVER

... To be done only if your Excel Software does not have it already installed
Step 0: Install Solver

- Windows: https://www.youtube.com/watch?v=g7C3XXyMV4A
- MacOS: https://www.youtube.com/watch?v=g7C3XXyMV4A
STEP 1: INPUT INSTANCE AND SOLVE
Example 2: Formulation

- Tesla makes two models of cars
  - Model I: makes a profit of $3 million per batch
  - Model II: sells for $5 million per batch

- Tesla has three plants with limited working hours
  - Plant 1: Frame I
    - at most 4 working hours per week
    - 1 hour to prepare a batch of Frame I
  - Plant 2: Frame II
    - at most 12 working hours per week
    - 2 hours to prepare a batch of Frame II
  - Plant 3: Assembly
    - at most 18 working hours per week
    - 3 hours to assemble a batch of Model I (using Frame I) and 2 hours to assemble a batch of Model II (using Frame II)

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Availability</th>
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<tbody>
<tr>
<td>$P_1$</td>
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<td>$P_2$</td>
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<td>$P_3$</td>
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</tr>
<tr>
<td>Profit</td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Question: What is the best product mix?

$$\text{max } Z = 3x_1 + 5x_2 \quad \text{(profit)}$$

- $x_1 \leq 4$ (hour constraint for plant 1)
- $2x_2 \leq 12$ (hour constraint for plant 2)
- $3x_1 + 2x_2 \leq 18$ (hour constraint for plant 3)
- $x_1 \geq 0$ (non-negative amount of commodity 1)
- $x_2 \geq 0$ (non-negative amount of commodity 2)
### Step 1.1.1: Input data

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<th>B</th>
<th>C</th>
<th>D</th>
</tr>
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<td></td>
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<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Number of working hours at Plant 1</td>
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<td>4</td>
</tr>
<tr>
<td>Number of working hours at Plant 2</td>
<td>0</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Number of working hours at Plant 3</td>
<td>3</td>
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<td>18</td>
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<tr>
<td>Profit (in millions)</td>
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### Step 1.1.2: Declare variables and objective

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<th>C</th>
<th>D</th>
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</tr>
<tr>
<td>3</td>
<td></td>
<td><strong>Model I</strong></td>
<td><strong>Model II</strong></td>
<td><strong>Availability</strong></td>
</tr>
<tr>
<td>4</td>
<td>Number of working hours at Plant 1</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>Number of working hours at Plant 2</td>
<td>0</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>Number of working hours at Plant 3</td>
<td>3</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>Profit (in millions)</td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

**Decision Variables**
- Number of batches of model I, $x_1$
- Number of batches of model II, $x_2$

**Objective**
- Total profit

- Profit cell B14 is a changing cell
- It is defined as an excel function “=B7*B10+C7*B11”
  - It represents $3x_1 + 5x_2$
Step 1.1.3: Declare constraints

- LHS are changing cells: they are the LHS of constraints
- Examples:
  - B17 cell is defined as an excel function “=B4*B10+C4*B11”
    - It represents $1 \times x_1 + 0 \times x_2$
  - B18 cell is defined as an excel function “=B5*B10+C5*B11”
    - It represents $0 \times x_1 + 2 \times x_2$

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tesla Production Problem</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
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**Decision Variables**

- Number of batches of model I, $x_1$
- Number of batches of model II, $x_2$

**Objective**

- Total profit

<table>
<thead>
<tr>
<th>A</th>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of working hours at Plant 1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Number of working hours at Plant 2</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Number of working hours at Plant 3</td>
<td>0</td>
<td>18</td>
</tr>
</tbody>
</table>
Step 1.2: Setup Solver

• Step 2.1: Open Solver Dialog
  • Data -> Solver

• Step 2.2: Setup Solver
  • Select the cell that represents the “Objective”
    • Choose objective cell B14
  • Check “Max” to indicate maximization
  • Select the cells that represent the “Variables”
    • Choose B10:B11 (press “Shift” key to select many cells)
  • Select the cells that represent the “Constraints”
    • Constraints can only involve two adjacent columns
    • Choose B17-B19 and C17-C19
  • Check “Make Unconstrained Variables Non-Negative” (if needed)
  • Select “Simplex LP” to tell the Solver that this is an LP
  • Click ”Solve”
Step 1.2: Setup Solver

- **Set Objective:** SBS14
- **Max**
- **Subject to the Constraints:** SBS17:SBS19 <= SCS17:SCS19

- **Make Unconstrained Variables Non-Negative**
- **Select a Solving Method:** Simplex LP
The constraint

$$B_{17}^\text{}:B_{19}^\text{} <= C_{17}^\text{}:C_{19}^\text{}$$

is the same as the system of constraints:

$$B_{17} <= C_{17}$$
$$B_{18} <= C_{18}$$
$$B_{19} <= C_{19}$$

But much more convenient to use!
Step 1.3: Solve (and get reports)

<table>
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<td></td>
</tr>
</tbody>
</table>

**Decision Variables**
- Number of batches of model I, x_1: 2
- Number of batches of model II, x_2: 6

**Objective**
- Total profit: 36

**Constraints**
- Number of working hours at Plant 1: LHS = 2, RHS = 4
- Number of working hours at Plant 2: LHS = 12, RHS = 12
- Number of working hours at Plant 3: LHS = 18, RHS = 18

After clicking solve, see two changes:

- **Change 1**: values in “Decision Variables”, “Objective”, and LHS cells will change
- Optimum decision variable values: (2,6)
- Optimum objective value: 36
Step 1.3: Solve (and get reports)

After clicking solve, see two changes:

- **Change 2:** solver window becomes

![Solver Results dialog box]

- There are 3 possible reports. Each one will be a separate tab in the excel file.
STEP 2: INTERPRET REPORTS

1. Answer Report
2. Sensitivity Report
3. Limits Report: No useful information